Teichmüller theory and moduli problems
| Vol. 1: | Number theory |
| Vol. 2: | The Riemann zeta function and related themes |
| Vol. 3: | Formal language aspects of natural computing |
| Vol. 4: | Commutative algebra and combinatorics |
| Vol. 5: | Convexity in discrete structures |
| Vol. 6: | Number theory and discrete geometry |
| Vol. 7: | Discrete mathematics |
| Vol. 8: | Lectures on operator theory |
| Vol. 9: | Essays on geometric group theory |
| Vol. 10: | Teichmüller theory and moduli problems |
Teichmüller theory and moduli problems

Proceedings of the U.S.-India workshop in Teichmüller theory and moduli problems, Harish-Chandra Research Institute, Allahabad, January 2006

Volume editors

Indranil Biswas
Ravi S. Kulkarni
Sudeb Mitra
Members of the Advisory Board

R. Balasubramanian (IMSc, Chennai, India)
R. B. Bapat (ISI, Delhi, India)
Manjul Bhargava (Princeton U, NJ, USA)
J. H. Coates (Cambridge, U, UK)
W. Goldman (U of Maryland, Md, USA)
G. Misra (IISC, Bangalore, India)
V. Kumar Murty (U of Toronto, Canada)
M. S. Narasimhan (IISC, Bangalore, India)
Nitin Nitsure (TIFR, Mumbai, India)
Gopal Prasad (U of Michigan, Michigan, USA)
M. S. Raghunathan (TIFR, Mumbai, India)
S. S. Sane (U of Mumbai, India)
V. D. Sharma (IIT (Bombay), Mumbai, India)
Alladi Sitaram (Formerly at ISI (Bangalore), India)
V. Srinivas (TIFR, Mumbai, India)
S. Thangavelu (IISC, Bangalore, India)
V. S. Varadarajan (UCLA, California, USA)
S. R. S. Varadhan (Courant Institute, New Cork, USA)
S. T. Yau (Harvard U, Mass, USA)
Contents

Members of the Advisory Board iii

Introduction
   Indranil Biswas, Ravi S. Kulkarni, Sudeb Mitra 1–3

Teichmüller Spaces as Complex Manifolds: Warwick 1992
   Clifford J. Earle 5–33

Moduli
   C. S. Seshadri 35–39

On the Divergence of Certain Geodesic Rays in Teichmüller Space
   Abdelhadi Belkhirat 41–51

On the Non-Quasi-Isometry of the Teichmüller Metric and Thurston’s
   Weak Metric
   Abdelhadi Belkhirat 53–60

Notes on Vector Bundles on Curves
   Usha N. Bhosle and Indranil Biswas 61–93

Moduli Spaces of Connections on a Riemann Surface
   Indranil Biswas and Vicente Muñoz 95–111

Teichmüller Geodesics, Delaunay Trinagulations, and Veech Groups
   Joshua P. Bowman 113–129

Introductory Bumponomics: The Topology of Deformation Spaces of
   Hyperbolic 3-Manifolds
   Richard P. Canary 131–150

A Short Note on Nonemptyness of Parabolic Moduli
   Arijit Dey 151–166

Limit Set of Quasiconformal Mapping Class Group on Asymptotic
   Teichmüller Space
   Ege Fujikawa 167–178

Finite Earthquakes and the Associahedron
   Frederick P. Gardiner and Jun Hu 179–194
# Contents

A Short Course on Teichmüller’s Theorem  
*F. P. Gardiner and Jun Hu*  
195–228

Prime Order Automorphism of Riemann Surfaces  
*Jane Gilman*  
229–246

On Complex Curves and Complex Surfaces Defined Over Number Fields  
*Ernesto Girondo and Gabino González-Díez*  
247–280

Moduli Spaces for Principal Bundles in Large Characteristics  
*Tomás L. Gómez, Adrian Langer, Alexander H. W. Schmitt and Ignacio Sols*  
281–371

Some Remarks About Curves on K3 Surfaces  
*Mihai Halic*  
373–385

The Moduli Stack of Vector Bundles on a Curve  
*Norbert Hoffman*  
387–394

The Klein-Maskit Combination Theorems  
*John Hubbard*  
395–412

Function Models for Teichmüller Spaces and Dual Geometric Gibbs Type Measure Theory for Circle Dynamics  
*Yunping Jiang*  
413–435

Stable Maps into the Classifying Space of the General Linear Group  
*Ivan Kausz*  
437–449

Limit Points of Iterated Function Systems  
*Linda Keen and Nikola Lakic*  
451–463

SaddleDrop: A Tool for Studying Dynamics in $\mathbb{C}^2$  
*Sarah C. Koch*  
465–479

The Action of Elliptic Modular Transformations on Asymptotic Teichmüller Spaces  
*Katsuhiko Matsuzaki*  
481–488

Cannon-Thurston Maps and Bounded Geometry  
*Mahan Mj*  
489–511

Computing a Generating Set of Arithmetic Kleinian Groups  
*Gregory Muller*  
513–517
Contents

On the Spectrum of Asymptotic Slopes
   A. J. Parameswaran and S. Subramanian  519–528

Rigidity, Past and Present  Chris Peters  529–548

On the Art of Calculating Accessory Parameters of Conformal Mappings of Circular Arc Polygons—General Considerations and Special Situations
   R. Michael Porter  549–576

Denjoy-Wolf Theorem on Riemann Surfaces  Hiroshige Shiga  577–582

Cardioids and Teichmüller Spaces  Toshiyuki Sugawa  583–596

Stratifying the Space of Moduli  Anthony Weaver  597–618
Introduction

The Harish-Chandra Research Institute (HRI), Allahabad, India, hosted the “US-India Workshop: Teichmüller Theory and Moduli Problems” from January 5, 2006 to January 15, 2006. This was the culmination of the “Year in Teichmüller Theory and Moduli Problems.” This year-long program was inaugurated in January 2005 by Clifford J. Earle (Cornell University, USA), and Frederick P. Gardiner (City University of New York, USA). They visited HRI during January – February, 2005, and gave a series of introductory lectures on Teichmüller theory. Clifford Earle gave a series of lectures on “Teichmüller theory, past, present, and future.” Frederick Gardiner gave some lectures on “A short course on Teichmüller’s theorem.” These lectures were followed by some survey lectures on “Moduli spaces of vector bundles” by M. S. Narasimhan in March 2005. William J. Harvey of King’s College, London, visited HRI in March 2005 and lectured on “Mapping Class Groups.” He surveyed some recent work about these groups which play a pivotal role in low-dimensional topology and geometry of surfaces. John H. Hubbard (Cornell University, USA) visited HRI in August 2005 and gave a course of lectures on “Holomorphic Dynamics.”

The principal aim of this Workshop was to bring closer two mathematical traditions, namely the Ahlfors-Bers School of the complex analytic approach to Teichmüller theory that has a strong presence in USA, and the algebro-geometric tradition of the moduli of vector bundles that has developed in India. The idea was to connect scholars from different countries with similar research interests, as an important step towards furthering joint research. The larger objective was to foster increased cooperation between the Indian, American and European scientific communities, working in various areas of moduli problems.

Teichmüller theory originated as a specialized branch of complex analysis. Over the last 30 years, it has blossomed into a big and booming field of research, interacting with many other areas of mathematics, like topology, geometry, and dynamics. A natural question in the study of Riemann surfaces is to parametrize the space of conformal structures on a given topological surface. Riemann mapping theorem already says that there is a unique conformal structure on the Riemann sphere, complex plane and the unit disk, i.e any Riemann surface structure on a simply connected domain is equivalent to one of these three cases. There is a well-studied 1-complex parameter family of Riemann surface structure on a compact surface of genus 1 (elliptic curves).
Riemann asserted that, for a compact Riemann surface of genus $g \geq 2$, the space $M_g$ of distinct conformal structures has complex dimension $3g - 3$. The space $M_g$ is called the Riemann’s moduli space. The algebraic geometers have studied this space extensively. During the late 1930s, Teichmüller followed an analytic approach, using quasiconformal mappings, and constructed a new space $T_g$, now called the Teichmüller space. The Teichmüller space has a canonical complex structure. The automorphism group of this structure is $\Gamma_g$, the Teichmüller Modular Group, which is isomorphic the outer automorphism group of the fundamental group of a compact surface of genus $g$. The quotient space $T_g / \Gamma_g$ can be canonically identified with $M_g$.

Broadly speaking, there are three main approaches to the study of moduli of Riemann surfaces and their higher-dimensional analogues:

(i) The analytic approach was initiated by Teichmüller and was further developed by Ahlfors, Bers, and their students and followers. An important aspect of this approach is that it can also be extended to noncompact surfaces including those of infinite type. The methods of quasiconformal mappings and Teichmüller theory also found interesting applications in the study of the dynamics of rational maps. Fundamental work in this area was done by Douady, Hubbard, McMullen, Sullivan, Thurston, and others during the 1980s.

(ii) The algebro-geometric approach was developed by Grothendieck, Deligne, Mumford, and many others.

(iii) The topological and hyperbolic/conformal geometric approach was initiated by Nielsen and Fenchel in the 1930s. Its importance for the study of 3-manifolds and Kleinian groups was recognized by Kulkarni, Marden, Maskit, and was revolutionized by Thurston, and later developed in fundamental ways by McMullen, Penner, Sullivan, and others in the 1980s.

As is well-known, Teichmüller theory has found interesting applications in modern particle physics. In the branch of physics known as String theory, elementary particles are modelled by loops of strings that generate Riemann surfaces as they move through time. These Riemann surfaces have physical interpretations, and Teichmüller theory can be used to study how they vary.

The mapping class group of a genus $g$ surface acts on $T_g$, and the quotient $M_g$, as noted before, parametrizes conformal structures on that surface. We can have interesting parameter spaces for many other structures and objects other than the conformal structure. For example, we can have moduli spaces of holomorphic vector bundles on a given base, moduli space of holomorphic maps from surfaces of fixed genus to a given projective variety etc. We refer to the article of Seshadri (in this volume) for a more technical introduction for the moduli spaces.

This workshop was funded by the National Science Foundation, USA, Department of Science and Technology, India, and the Infosys Foundations. There were speakers from USA, UK, Spain, Japan, China, France, Germany, Switzerland, Mexico, Sultanate of Oman, and India. The lectures covered a remarkably broad range, helping the younger participants to get an exposure to current research areas. There was also a special event – John Hubbard (USA), Richard Canary (USA), and
S. Ramanan (India) gave general survey lectures on the historical developments and various ramifications of Teichmüller theory, hyperbolic geometry, 3-manifolds, and moduli of vector bundles.

The editors wish to thank the graduate students and the staff at HRI who worked very hard to make this workshop a success. Many thanks to Professor Amitabha Raychowdhury (Director, HRI), and the graduate students Vikram Aithal, Kuntal Chatterjee, Krishnendu Gangopadhyay, and Siddhartha Sarkar. Special thanks to Prof. Madhav Modak, who took pains to go through every paper, and corrected many typos and ambiguities.

Indranil Biswas, Ravi S. Kulkarni, Sudeb Mitra