

# Mathematics and machine learning program

MICHAEL R. DOUGLAS

This special issue of *Advances in Theoretical and Mathematical Physics* (ATMP) is devoted to the Program on Mathematics and Machine Learning held at the Center of Mathematical Sciences and Applications at Harvard University from September 3 through November 1, 2024. The program was co-organized by François Charton, Michael R. Douglas, Mike Freedman, Fabian Ruehle and Geordie Williamson, and welcomed over 80 participants from many subfields of mathematics, physics and computer science. For eight weeks we explored the new opportunities created by applying the most recent developments in machine learning to mathematical problems old and new, proposed problems and formed working groups, and began in-depth studies. Our work continued after the program, and many of the results are reported in the articles here.

Let us briefly outline the contents by subtopic. First come papers on methods and new software developed specifically for mathematical applications. A noteworthy feature of the program was the close collaboration between mathematical and machine learning experts, and much was learned on both sides. These papers include “Int2int — a Transformer Model for Integer Sequences” by Charton on a new transformer model and “Generative Modeling for Mathematical Discovery” by Sutherland *et al.* on a new implementation of the `funsearch` method, “Merging Hazy Sets with m-Schemes: A Geometric Approach to Data Visualization” by Barth *et al.*, “Kolmogorov-Arnold stability” by Dzhenzher and Freedman, and “Mathematical Data Science” by Douglas and Lee, which surveyed this broad area with case studies such as the discovery of murmurations.

Going the other direction, there were many talks and discussions on studying machine learning using ideas and methods from mathematics and physics. This topic is represented in the issue by “Two-Point Deterministic Equivalence for Stochastic Gradient Dynamics in Linear Models” by Atanasov *et al.*, and by work to appear in later issues of ATMP.

Two program weeks focused on number theory, leading to many papers: “Learning Euler factors of elliptic curves” by Babei *et al.*, “Machine Learning Approaches to the Shafarevich-Tate Group of Elliptic Curves” by Banwait

*et al.*, “Machine learning the vanishing order of rational  $L$ -functions” and “Learning Fricke signs from Maass Form Coefficients” by Bieri *et al.*, and “Studying number theory with deep learning: a case study with the Möbius and squarefree indicator functions” by Lowry-Duda.

A week on graph theory and combinatorics spawned many successful projects, leading to: “Advancing Geometry with AI: Multi-agent Generation of Polytopes” by Swirszcz *et al.*, “Learning the symmetric group: large from small” by Petschack *et al.*, “Reinforcement Learning the Chromatic Symmetric Function” by Bérczi and Klüver, “Interpretable Machine Learning for Kronecker Coefficients” by Butbaia *et al.*, and “Machine learning toric duality in brane tilings” by Capuzzo *et al.*

A week on knot theory and representation theory produced “On the Learnability of Knot Invariants: Representation, Predictability, and Neural Similarity” by Lindsay and Ruelle, and “CayleyPy RL: Pathfinding and Reinforcement Learning on Cayley Graphs” by Chervov *et al.*

All the participants agreed that beyond our specific projects and discoveries, we had seen the future of mathematics, a future in which researchers combine the power of traditional mathematical methods with new computational and statistical tools to see farther and dig deeper than before.

We thank the CMSA, its staff and director Dan Freed for hosting the program and making it run smoothly; Chuck Doran for the opportunity to publish this special issue; and ATMP and its staff for all of their help.

March 2025

CMSA, HARVARD UNIVERSITY  
CAMBRIDGE, MA 02138, USA  
*E-mail address:* `mdouglas@cmsa.fas.harvard.edu`