

PREFACE

The editors of the Asian Journal of Mathematics wished to honor Professor Heisuke Hironaka on the occasion of his 80th Birthday with a special issue of the journal. A group of guest editors was formed, who in turn solicited mathematical contributions which constitute this issue.

This is not the place to present Professor Hironaka or his work, and we refer to the Introduction of the RIMS publications volume¹ published on the occasion of his Kiju, or 77th Birthday. Let it suffice to say that the same spirit of admiration and friendship fueled the preparation of this volume.

Let us now briefly present the content of the papers gathered here.

1. The paper of Benito, Encinas and Villamayor is a presentation for non experts of the current understanding of the main ideas of Hironaka's proof of resolution in characteristic zero. It also describes the evolution of ideas and stresses the structural (or "natural") aspects of the proof.
2. The paper of Bierstone, Grigoriev, Milman and Włodarczyk analyses the process of resolution of singularities of an ideal from the viewpoint of complexity. The complexity of the resolution of an ideal on an m dimensional non singular variety is bounded by a function of the Grzegorzczak class \mathcal{E}^{m+3} , which means that its construction requires $m + 3$ nested primitive recursions.
3. The paper of Bierstone, Milman and Temkin proves that the desingularization algorithm for algebraic varieties in characteristic zero of the first two authors is functorial with respect to regular (and not just smooth) morphisms. The term \mathbf{Q} -universal comes from the fact that it is induced from its restriction to varieties defined over \mathbf{Q} , using the fact that every variety in characteristic zero admits a regular morphism to such a variety. It implies in particular that the desingularization algorithm extends functorially to localizations and henselizations of varieties.
4. The paper of Blanco and Encinas is a part of the efforts to prove resolution in positive characteristic. Several authors have used algebras graded by rationals (for example \mathbf{Q} -Rees algebras) to encode data related to log-resolution of ideals. This paper offers a comparison of these approaches and shows the working of the authors' elimination method to provide a resolution algorithm for these algebras in characteristic zero, which gives log-resolution for ideals.
5. The paper of Brasselet and Teissier studies "integration in the fibers" of subanalytic continuous differential forms in the context of triangulable subanalytic maps. One of its tools is a "relative resolution of singularities" for a simplicial map, which permits the definition of relative Whitney forms in this framework, and leads to the proof of existence of a subanalytic continuous relative primitive for a subanalytic continuous differential form which is exact in the non singular fibers.
6. The paper of Bravo and Villamayor is an introduction to the method of Rees algebras and elimination for resolution of singularities. The geometric idea is to replace the idealistic trace of the singular space at a point on a non singular hypersurface of maximal contact by a local projection and an

¹Publications of the R.I.M.S. Kyoto, Vol.44, Issue 2, 2008.

elimination procedure, which exist in all characteristic. It is recommended to read this paper before that of Blanco and Encinas.

7. The paper of Cossart presents the main ideas of the proof of Cossart-Piltant of resolution of singularities of a quasi-projective scheme of dimension three of a field of positive characteristic which is differentially finite over a perfect field. It contains a thorough analysis of several important examples, which display phenomena which occur in characteristic p and are contrary to the "usual" behavior of resolution invariants in characteristic zero. The author also shows that there are in general no spaces satisfying some modifications in positive characteristic of the concept of maximal contact.
8. The paper of Cutkosky is a presentation, simplification and analysis of Abhyankar's celebrated proof of resolution of singularities of surfaces in positive characteristic p . It makes no sense to try to summarize it but the original papers are so fundamental and difficult that we are certain that many people will be grateful to the author for this "skeleton key".
9. The paper of Hauser and Schicho is a list of problems on singularities with comments, which contains in particular problems which can be helpful for an introduction to some of the mathematics involved in resolution of singularities, and some others whose solution would help the resolution itself.
10. The paper of Kollár presents applications to simultaneous normalization of the fibers of an algebraic map of the notion of an algebra husk with respect to a morphism. In fact it applies not only to normalization itself but also to many normalization-like operations. The paper also contains a short proof of a generalization of the numerical criterion for simultaneous normalization of Chiang-Hsieh and Lipman.
11. The paper of Tarrío, López and Lipman is a very thorough presentation of the categorical framework needed to establish a bivariant theory in the context of schemes by means of Grothendieck duality. It contains a theory of Hochschild (co)homology for schemes that are flat, separated and essentially of finite type over a noetherian scheme S and paves the way for the construction of relative fundamental class maps in bivariant Hochschild theory.

It is appropriate that many papers in this volume present aspects of resolution inspired by the search of a proof in positive characteristic, since Hironaka himself is working with determination in this direction. The others represent tributes of a more general nature to a great mathematician, teacher and friend whose work is outstanding both in its depth and in the variety of its consequences. The editors and guest editors hope that the content of this volume will inspire work in singularity theory.

Guest Editors:

Lê Dũng Tráng and Bernard Teissier