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A review of image denoising methods

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Image denoising is a fundamental and important task in the field of digital image processing and computer vision. Image noise concatenation inevitably occurs during image acquisition and transmission, which leads to the degradation of image quality. The presence of noise has some negative effects on various practical applications such as object recognition, medical image analysis, and hyperspectral remote sensing. A lot of research work has provided a solution to this problem, and many methods have been developed in the literature. This paper focuses on classifying some representative works in the field of image denoising, and provides a brief review with several promising directions for further investigation in the future.

1. Introduction

Image denoising, as an essential step to improve image quality, has always been a hot research topic. In today's information age, images as an important way of information transmission are closely related to people's lives. All applications of images can only meet the requirements of production and life on the premise of high-quality images. In the process of image acquisition, storage, and transmission, however, noise can be generated due to the interference of external factors, which leads to low-quality images. Therefore, the research on image denoising is particularly important. The purpose of image denoising is to remove or suppress noise from images concatenated by noise, and restore the original images as accurately as possible, while retaining fine details such as edges and textures. Since noise, edges and texture are to high frequency components, it is difficult to distinguish them in the process of denoising. From numerous types of noise prevailing in different kinds of images, additive white Gaussian noise (AWGN), impulse noise (also known as salt and pepper), quantization noise, Poisson noise and speckle noise are the most frequently discussed noises in the literature [1]. AWGN primarily occurs in analog circuits during image acquisition and transmission. The prevalence of other types of noise such as quantization noise, impulse noise, speckle noise, and Poisson noise occur mainly due to faulty manufacturing, bit errors, and inadequate photon counting during image acquisition. In general, how to recover meaningful information from noisy images in the process of noise reduction and obtain high quality images is a vital problem. Although great progresses in image denoising have been made in the past two decades [2, 3, 4, 5], the demand for high-performance image denoising methods is still urgent in practical applications. The main reason is that, from mathematical view, the nature of image denoising is an inverse problem and the solution is not unique. Therefore, the key issue is how to model the prior of images and derive the optimal solution under the prior constraints. In the following sections, we will briefly introduce different image denoising techniques.

1.1. Problem statement

Mathematically, the noisy image corrupted by AWGN can be modeled as follows:

$$y = x + n, \tag{1}$$

where y denotes the observed noisy image, x is the unknown clean image, and n represents the AWGN that is assumed to be independent and identically distributed with zero mean and standard deviation σ_n. It is the simplest way to model a noisy image. There are other models for Poisson noise and impulse noise. In CCD (charged coupled devices) cameras noise may be present in the electron circuitry (thermal noise), due to inadequate photon count (photon noise) or it may be quantization noise. However, AWGN is the most common encountered noise in real-time applications, hence the AWGN model is the primary focus of this paper. In practical applications, AWGN with the standard deviation σ_n (i.e., noise level) can be estimated by various methods such as median absolute deviation (MAD) [6], block-based estimation method [7], and PCA-based method [8]. The goal of the image denoising procedure is to suppress the noise from natural images while minimizing the loss of features in the denoised images, and improving the visual quality and the signal-to-noise ratio (SNR) of images. The major challenge of image denoising is how to effectively distinguish noise and texture details from an observed noisy image.

1.2. Noise suppression

In fact, achieving a clean image x from the Eq. (1) is an ill-posed problem, which means that we cannot obtain a unique solution from the additive noise model. In the last two decades, researchers continuously thrive to develop efficient denoising algorithms which aiming to recover a reasonable estimate $\hat{x}$ from the observed noisy image y, while preserving its fine features and edges. There exist a lot of image denoising algorithms in the literature.

On the basis of the theories and techniques adopted by denoising algorithms, they can broadly be classified into the following categories: spatial domain filtering that depends on the local/non-local spatial correlation of pixels, transform domain filtering that employs the energy compact of image transforms, sparse representation based methods that are based on learned representation dictionaries, low-rank based methods that adopt adaptive dictionaries derived by the noisy image itself, and deep learning based methods that use deep convolutional neural networks to learn a denoiser.

The rest of this paper is organized as follows: Section 2 introduces the various spatial domain filtering methods. The transform domain filtering methods and the well-known block-matching and 3D filtering (BM3D) are discussed in Section 3. The sparse representation and dictionary learning methods are followed in Section 4. Sections 5 and 6 further discuss several low-rank based methods and deep learning-based methods. A summary and some possible future research directions are reached in Section 7.

2. Spatial domain filtering

Ideally, filtering is supposed to be the primary solution to image noise removal, that is, suppression of the unwanted variation in pixel intensity values. In fact, signal filtering is fundamental to basic image processing and has long been used for smoothing, sharpening, and edge detection. Spatial domain filtering is to remove the image noise by directly operating the pixels in the neighborhood [12, 11, 13, 14, 15, 10, 16, 17, 18, 19, 20, 21, 22, 23, 24]. Spatial domain filtering [25, 26, 27, 28] directly estimates each pixel (or region) of the latent clean image with its neighboring or similar pixels (or regions) of the noisy observation in some way. Contrastively, transform domain filtering methods [29, 30, 31] first project the noisy image into a set of bases, then apply shrinkage rules to the transformed coefficients, and finally obtain the denoised image by the inverse transform. According to the manner of selecting pixel candidates used in the filtering process, the spatial filters can be divided into local filters and non-local filters. The underlying principle of this kind of denoising algorithms is that noise is uncorrelated amongst pixels, while the intensity of real signal pixels are correlated with each other [32, 33].

2.1. Local filters

A spatial filter is considered to be local if its filter support for denoising a pixel is restricted by spatial distance [13, 14, 15, 10, 16, 17, 18, 19, 20, 23, 24]. It is different from the non-local filter that exploits the correlation between the entire range of pixels in an image. The most popular local filters designed for noise reduction are the mean filter and the median filter. The mean filtering [17] calculates the central pixel by convolution and the average of the pixels in the template. Since the mean value of Gaussian noise is 0, and the gray value of the center point is similar to that of other pixels in the template, so the averaging operation can reduce the noise. Usually, this kind of denoising methods is suitable for suppressing Gaussian noise. Unfortunately, the average value often smoothes out the high frequency information of the image, so that the edges and textures of the denoised image can be over-smoothed. Unlike the mean filtering, the median filtering is to sort the pixels in the template and to replace the gray value of the center point with the middle value of the sorted pixels, so as to remove the image noise. This method is useful for removing salt and pepper noise, but it is not for removing Gaussian noise. It is easy to lose edge information and affect the visual effect of the image.

To overcome these limitations aforementioned, Gaussian filtering (GF) [34] and Wiener filtering (WF) [19, 20] are further introduced, which do not employ mean of neighboring pixels. GF is a typical linear filter, which belongs to the category of local filters and has been used in image denoising for a long time, while WF is a class of optimum linear filters which involve linear estimation of a desired signal sequence from another related sequence. Most local denoising filters have primarily been the improvements of Gaussian filtering, which have been proposed to provide better edge preserving ability [35].

2.2. Non-local filters

In [36], Buades et al. introduce a filter based on experimental methodology to exploit similarity amongst pixels in a non-local manner, popularly known as non-local mean (NLM) filter. This pioneering work clearly established that the self-similarity amongst characteristics of an image in a non-local manner is the biggest potential basis in the field of image denoising. NLM filter exploits presence of similar features or patterns in the image.

Its basic idea is that building a pointwise estimation of the noisy image where each pixel is obtained as a weighted average of pixels centered at regions that are similar to the region centered at the estimated pixel. For a given pixel x_i in an image x, its NLM-filtered new intensity value is denoted by $NLM(x_i)$. Specifically, let x_i and x_j be the square patch centered at pixels x_i and x_j, respectively. The weight of patch x_j to patch x_i, denoted as $w_{i,j}$, is calculated by the Gaussian L2-distance between the patches centered at the above central pixels, which can be formulated by

$$w_{i,j} = \frac{1}{c_i} \exp\left(- \frac{\|x_i - x_j\|_2^2}{h} \right), \tag{2}$$

where c_i is the normalization factor, and h is the filter parameter that controls the decay rate of the exponential function. Different from local filtering methods, NLM can make full use of the information provided by the given image, which can lead to a robust estimation of the noisy image. Since then, various improved versions of NLM have been proposed in the literature. Some works of these improvements take efforts to accelerate the NLM denoising algorithm [37, 38, 39, 40, 41, 42], while others focus on how to enhance the performance of the algorithm [43, 44].

3. Transform domain fitering

Image denoising methods have gradually developed from the initial spatial domain filtering to the present transform domain filtering. Initially, transform domain methods were developed from the Fourier transform, and then a variety of transform domain methods have gradually emerged, such as cosine transform, wavelet domain methods, multiscale geometric transforms, and hybrid transforms used in BM3D [45]. Transform domain methods depend upon the following observations: the frequency characteristics of image information and noise are significantly different in the transform domain, and noise is easily distinguished from the image information.

3.1. Classical transform filtering

In contrast to the denoising methods in the spatial domain, transform domain methods exploit the property of sparsity, which means that the signal can be represented by fewer number of non-zero transform coefficients or an image could be represented as a linear expansion of a few high valued coefficients. This property has made them an extremely applicable digital signal processing tool both in 1-D and 2-D domain. The representation attributes like localization, isotropy, multi-resolution and the orientation of basis functions at a variety of directions are the major properties of image transform methods. There are a large number of variations available in this category such as fast Fourier transform, discrete cosine transform, wavelet transform, and curvelet transform.

The most commonly used transform in denoising is the wavelet transform [46], where the input data is decomposed into its scale-space representation. It has been proved that the use of wavelets successfully removes noise while preserving the image fine details, regardless of its frequency content [47, 48, 49, 50, 51, 52]. Since the wavelet transform has many good characteristics, such as sparsity, multi-scale representation, and low computational complexity, it is still active in image denoising nowadays [53]. However, the wavelet transform adopts the fixed basis functions, e.g, Haar wavelets and Daubechies wavelts. Therefore, the performance of wavelet-based denoising methods largely depends on the selection of wavelet bases. If it is not selected properly, the image shown in wavelet domain cannot be well represented, resulting in poor denoising performance.

3.2. Hybrid transform filtering

As an effective and powerful extension of NLM approach, BM3D that employs non-local self-similarity of the image and the hybrid transforms is the current state-of-the-art method for image denoising, which was proposed by Dabov et al. [45]. BM3D is a two-stage nonlocally collaborative filtering method in the hybrid transform domain. In this method, similar patches are stacked into 3D groups by block matching, and the 3D groups are transformed into wavelet and cosine transform domain. Then the hard thresholding or Wiener filtering with coefficients is employed in the transform domain. Finally, all estimated patches after an inverse transform of coefficients are aggregated to reconstruct the whole denoised image. However, when the noise increases gradually, the denoising performance of BM3D decreases greatly and artifacts are introduced especially in flat areas.

In order to improve the denoising performance, many improved variations of BM3D have been developed [54, 55]. For example, Maggioni et al. [55] recently proposed a BM4D denoising method, which providing the extension of BM3D to volumetric data. It utilizes cubes of voxels, which are stacked into a 4-D group. The 4-D transform applied to the group simultaneously exploits the local correlation and the nonlocal correlation of voxels. Thus, the spectrum of the group is highly sparse, leading to very effective separation of signal and noise by using some coefficient shrinkage techniques.

4. Sparse representation based denoising

The redundant and sparse representations have been the basic dynamic foundation for researchers in image denoising. The goal of sparse representation is to learn a set of basis functions that can adaptively represent images and inherit the energy compact property of the wavelet transform. To this end, various dictionary learning methods for sparse representation have been designed [56]. As analyzed in [57], image denoising based on sparse representation can be formulated as the following minimization problem,

$$\hat{x} = \arg\min_{x} \frac{1}{2}\|y - x\|_2^2 + \lambda R(x), \tag{3}$$

where $\|y - x\|_2^2$ denotes the data fidelity term that indicates the difference between the original image and the noisy image, $R(x)$ denotes the regularization term, and $\lambda > 0$ is the regularization parameter. Sparse representation assumes that a patch x_i can be sparsely represented by a linear combination of atoms in a redundant dictionary D, i.e., $x_i = D\alpha_i$, where α_i is the sparse codes.

As a dictionary learning method, the sparse representation model can not only be learned from a representative data set, but also even from the noisy image itself. The K-SVD algorithm [58, 59] is one of the most popular and powerful numerical methods for tackling the underlying energy minimization problem. The basic idea behind K-SVD denoising is to learn the dictionary D from noisy image y by solving the following joint optimization problem

$$\arg\min_{x,D,\alpha} \lambda\|y - x\|_2^2 + \sum_i \|R_i x - D\alpha_i\|_2^2 + \sum_i \mu_i \|\alpha_i\|_1, \tag{4}$$

where R_i is the matrix extracting patch x_i from image x at location i, and α_i are sparse representation coefficients. Eq. (4) can be solved by iterative optimization. More specifically, each iteration comprises of employing orthogonal matching pursuit (OMP) to estimate the coefficients for each patch (an initial dictionary is used for computing sparse approximations of all patches) and updating the dictionary using singular value decomposition for one column at a time. Each patch can be estimated from a series of patches from the dictionary. K-SVD simulated the era of denoising with learned dictionaries, which restores image information using a more adaptive model. However, this method still needs improvement in case of large patches due to its high computational burden and limited size of the dictionary [59].

The learned simultaneous sparse coding (LSSC) is quite similar to K-SVD where similar patches are denoised using the same sparse decomposition. This method exploits self-similarity criteria of images in a non-local manner [60]. In case of K-SVD, slight change in the input can lead to a significant change in dictionary atoms, which is undesirable. The motivation of LSSC is to address this limitation of K-SVD. Specifically, LSSC was proposed as an improvisation to K-SVD by exploiting similar patches in a non-local manner. This method certainly speeds up the process of searching atoms in an unstructured dictionary. It has been found that LSSC can provide the state-of-the-art denoising performance, but it has a high computational complexity. Although LSSC has achieved satisfactory denoising results by employing clustering in sparse decompositions, its performance largely depends upon an initial dictionary that has to be trained offline with high quality images. Unlike it, the convolutional sparse representation (CSR) denoising method can achieve much higher performance than LSSC along with lower complexity [29].

In addition, the nonlocal centralized sparse representation (NCSR) model [61] naturally integrates nonlocal self-similarity (NSS) into the sparse representation framework. It is one of the most concerned image denoising method at present. As mentioned in [61], NCSR is very effective in reconstructing both smooth and the textured regions. Despite of successful combination of the above two techniques, the iterative dictionary learning and the nonlocal estimate of unknown sparse coefficients make this algorithm computationally expensive, which largely limits its applicability in many practical applications.

5. Low-rank denoising

Different from sparse representation, the low-rank based denoising model is to format the similar patches as a matrix with each column being a stretched patch vector, and exploit the low-rank prior of this patch matrix to suppress image noise [62, 63]. Low-rank denoising methods first appeared in field of matrix completion, and has made great progress under the drive of Candès and Ma [64]. In recent years, the low-rank model can achieve a good denoising performance so the researches of low-rank denoising methods have been gradually thorough.

Many existing low-rank methods formulate image denoising as a general low-rank matrix approximation (LRMA) problem [65], which aims to recover the underlying low-rank matrix from its degraded observation. The existing LRMA methods can be generally divided into two categories: the low-rank matrix factorization (LRMF) based methods [66, 67, 68, 69, 70, 71, 72, 25, 75, 73, 74] and the rank minimization based methods [65, 76, 77, 78]. The LRMF based methods aims to factorize a given matrix into two smaller ones, ensuring that their product approximates the given matrix under certain fidelity loss functions. In fact, LRMF is basically a non-convex optimization problem. The rank minimization based methods are to reconstruct the underlying data matrix by imposing an additional rank constraint on the estimated matrix. As the rank minimization problem is computationally NP-hard due to the noncontinuous and nonconvex nature of the rank function, it is usually relaxed to convex or nonconvex regularizes [79].

The commonly used convex relaxation of the rank function is the nuclear norm minimization (NNM) [78]. Due to its convex property, the NNM problem can be easily solved with theoretical guarantees [78, 80] in a closed-form solution by singular-value thresholding [78]. Although NNM has been widely used for low-rank matrix approximation, it treats all singular values equally, which indicates that large singular values are more penalized heavily than small ones. This is unreasonable because different singular values may have different importance. Hence, singular values should be handled differently. In practical applications, the underlying matrix data may be corrupted without incoherence. In such cases, NNM-based methods often fail to achieve a good solution, and the results may deviate seriously from the ground truth.

In contrast, the weighted nuclear norm minimization (WNNM) [65, 76] is a common nonconvex relaxation of the rank function, which is to overcome the same penalization of different singular values. Essentially, WNNM is an extension of NNM and has a global closed-form solution when the weights are in a non-descending order. Given a weight vector w, the weighted nuclear norm proximal problem consists of finding an approximation x of y that minimizes the following cost function:

$$\hat{x} = \arg\min_{x} \|y - x\|_F^2 + \|x\|_{w,*}, \tag{5}$$

where $\|x\|_{w,*} = \sum \|w_i \sigma_i(x)\|_1$ is the weighted nuclear norm of x, σ_i denote the singular values of x. Here w_i is the weight assigned on singular value $\sigma_i(x)$. It is also shown in [65] that if the weights satisfy $0 \leq w_1 \leq \cdots \leq w_n$, then the problem (7) has a unique global minimum and can be obtain by the singular value decomposition of y, i.e.,

$$\hat{x} = U S_w(\Sigma) V^T, \tag{6}$$

where $S_w(\Sigma) = \max(\Sigma - \mathrm{diag}(w), 0)$ is a shrinkage operator of singular values, Σ is a diagonal matrix formed by the singular values, U and V are the right and left singular vector matrix of y, respectively.

In the denoising application of WNNM, the large singular values are shrunk less while the small ones are shrunk more to keep the faithful information of the underlying data. Compared with NNM, WNNM achieves better denoising performance in terms of both qualitative and quantitative evaluations. However, these low-rank denoising methods do not involve geometric modeling of local images, which resulting in unsatisfactory restoration of sharp edges and detailed structures.

6. Deep learning-based denoising

Although most of the aforementioned denoising methods have achieved reasonably good performance in image denoising, they suffer from the following drawbacks [81]: (1) optimization methods for the test phase, (2) manual setting of parameters, and (3) a certain model for single denoising task. Recently, owing to the flexible architectures, deep learning techniques have strong abilities to effectively overcome the drawbacks of these methods. Deep learning-based methods can adaptively learn image representation models from large training data, and then use the learned parametric models for image denoising. Such denoising methods can achieve very impressive results and high computational efficiency.

The recently developed deep learning methods are based on the classical deep convolutional neural network (DCNN) [82]. The general model of deep learning based denoising methods can be modeled by

$$\min_{\Theta} Loss(\hat{x}, x), \quad s.t. \quad \hat{x} = \mathcal{F}(y, \sigma; \Theta)$$

where $\mathcal{F}(\cdot)$ is a DCNN with a parameter set Θ, and $Loss(\cdot)$ is the loss function to measure the similarity between denoised image $\hat{x}$ and noise-free image x. Deep learning model for image denoising has been recently attracting considerable attention due to its favorable denoising performance.

In [83], Zhang et al. proposed a residual learning strategy of CNN for training acceleration, and proposed feed-forward denoising convolutional neural networks (DnCNN). DnCNN aims to learn a mapping function $\hat{x} = \mathcal{F}(y; \Theta_\sigma)$ between the input noisy observation y and the desired output $\hat{x}$. The model parameters Θ_σ are trained for noisy images corrupted by AWGN with a fixed noise level σ. Subsequently, to remove spatially variant noise by a single network, they presented a fast and flexible denoising CNN with a tunable noise level map [84]. To handle non-white, spatially dependent and anisotropic noise, Benou et al. [85] introduced a spatiotemporal denoising framework using deep neural networks. Yang et al. [86] presented a generative adversarial network (GAN) with Wasserstein distance and perceptual loss for image denoising. In [87], GAN is used to learn noise distribution and to produce the paired training data, which results in an effective method for blind denoising. Furthermore, Ran et al. [88] proposed a residual encoder decoder Wasserstein GAN for 3D image denoising. In addition, feature attention mechanism has been proven to be a way to improve the denoising performance [89]. The architecture of DCNN is very important issue for deep learning-based image denoising, which can be addressed by automatically searching effective network architectures [90]. However, these data-driven denoising methods require a large amount of training data, and often show significant performance degradation when there are discrepancies between the training data and the test data. But large training data are not always available in practice.

7. Summary

In this paper, an earnest effort has been made to classify and explain various image denoising methods. It can be seen that it is highly important to keep developing new techniques to remove noise in images. The basic idea of image denoising is to eliminate noise pixels while preserving edges. Extensive efforts by a large number of researchers have generated a structural literature which exhibit substantial progressive growth attained by a series of sequential incremental improvements. Although it is nearly impossible to cover all of them, we have covered each domain of image denoising with several representative methods for each category. These methods have been divided into five categories: spatial filtering, transform domain methods, sparse representation based methods, low-rank based methods, and deep learning-based methods. Figure 1 illustrates the relationship between different denoising methods. In general, the performance of denoising methods shown in this figure is increasing from left to right. We can conclude from the above approaches for image denoising that the usage of DCNN is to provide better results than the traditional filtering and model-based methods used before.

Most denoising methods tend to leave residual noise and cannot sustain their performance at higher noise levels. While image denoising for AWGN removal has been well-studied, little work has been done on real image denoising. Also, using deep learning techniques to learn features requires the ground truth. However, the obtained real noisy images do not have the ground truth. These challenges are very urgent to for future scholars. With this review, we hope to provide a better understanding of the work that has been done and help researchers looking to work in this field.

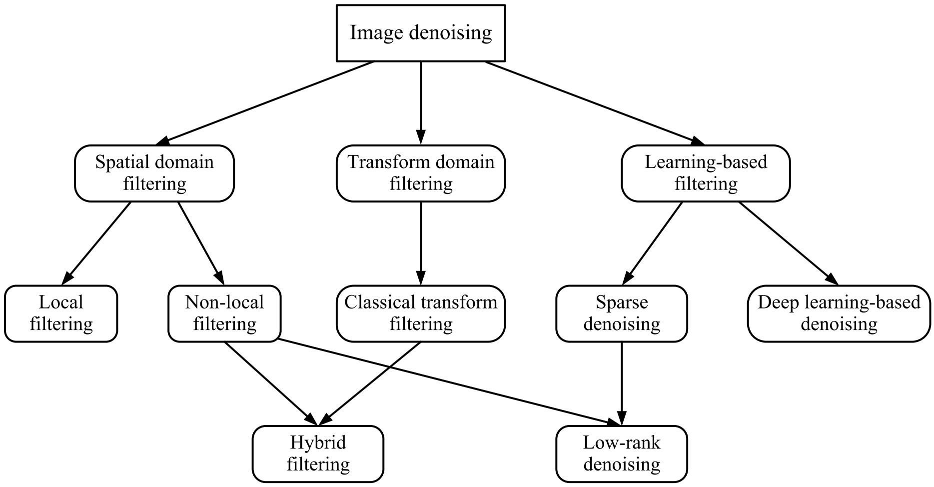

Figure 1: Relationship between different denoising methods.

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