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LONG TERM SPATIAL HOMOGENEITY FOR A CHEMOTAXIS MODEL WITH LOCAL SENSING AND CONSUMPTION*

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Abstract. Global weak solutions to a chemotaxis model with local sensing and consumption are shown to converge to spatially homogeneous steady states in the large time limit, when the motility is assumed to be positive and C^1-smooth on $[0,\infty)$. The result is valid in arbitrary space dimension $n\geq 1$ and extends a previous result which only deals with space dimensions $n\in\{1,2,3\}$.

Keywords. Convergence; Liapunov functional; chemotaxis-consumption model; local sensing.

AMS subject classifications. 35B40; 37L45; 35K51; 35Q92.

1. Introduction

Let Ω be a smooth bounded domain of $\mathbb{R}^n$, $n\geq 1$, and consider the initial boundary value problem

$$\partial_t u = \Delta(u\gamma(v)) \quad \text{in} \quad (0,\infty)\times\Omega, \tag{1.1a}$$

$$\partial_t v = \Delta v - uv \quad \text{in} \quad (0,\infty)\times\Omega, \tag{1.1b}$$

$$\nabla(u\gamma(v))\cdot\mathbf{n} = \nabla v\cdot\mathbf{n} = 0 \quad \text{on} \quad (0,\infty)\times\partial\Omega, \tag{1.1c}$$

$$(u,v)(0) = (u^{in},v^{in}) \quad \text{in} \quad \Omega, \tag{1.1d}$$

which describes the dynamics of a population of bacteria with non-negative density u and of a signal with non-negative concentration v. On the one hand, according to (1.1a), the diffusive motion of the bacteria is not only monitored by the signal through the motility function γ but also biased by a chemotactic effect generated by the signal. On the other hand, the signal is consumed by the bacteria, as reflected by the reaction term on the right-hand side of (1.1b). The latter mechanism is in sharp contrast with classical Keller-Segel chemotaxis models [4], in which the sink term $-uv$ in (1.1b) is replaced by $u-v$, so that bacteria produce the signal that alters their motion, see the survey articles [1–3, 9] and the references therein for a more precise account. Therefore, the dynamics of (1.1) is expected to differ significantly. A first hint in that direction is the following property: if (u_s,v_s) is a stationary solution to (1.1) with $u_s\not\equiv 0$, then necessarily $v_s\equiv 0$ by (1.1b). In that case, it readily follows from (1.1a) that $\gamma(0)u_s=$ const., which reduces to $u_s=$ const. when $\gamma(0)>0$. It is thus expected that the positivity of both $\gamma(0)$ and $\|u^{in}\|_1$ implies that any global non-negative solution (u,v) to (1.1) satisfies

$$\lim_{t\to\infty}(u(t),v(t)) = \left(\frac{\|u^{in}\|_1}{|\Omega|},0\right) \tag{1.2}$$

in an appropriate topology. That this convergence holds true in $L^\infty(\Omega,\mathbb{R}^2)$ is shown in [6, Theorem 1.2] when $\gamma\in C^3([0,\infty))$ is positive on $[0,\infty)$ with $\gamma'<0$ on $(0,\infty)$ and $\|v^{in}\|_\infty$ is sufficiently small and in [7] when $\gamma\in C^3([0,\infty))$ is positive on $[0,\infty)$ and

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the space dimension n ranges in $\{1,2,3\}$. The required regularity of γ is subsequently relaxed in [8, Theorem 1.2], where the validity of (1.2) is established under the sole assumption

$$\gamma \in C^1([0,\infty)), \quad \gamma > 0 \text{ on } [0,\infty), \tag{1.3}$$

still for $n\in\{1,2,3\}$, though in the weaker topology $H^1(\Omega)'\times L^\infty(\Omega)$. The main purpose of this note is to show that the assumption (1.3) is sufficient to prove that the convergence (1.2) holds true in arbitrary space dimension in $H^1(\Omega)'\times H^1(\Omega)$, see Theorem 1.1 below. We further deduce the convergence of v to zero in $L^\infty(\Omega)$ from Theorem 1.1, the regularizing effect of the heat semigroup, and the time monotonicity of $\|v\|_\infty$, see Corollary 1.1 below.

The statement of the main result of this note requires to introduce some notation: first, for $z\in H^1(\Omega)'$, we set $\langle z\rangle := \langle z,1\rangle_{(H^1)',H^1}/|\Omega|$ and note that

$$\langle z\rangle = \frac{1}{|\Omega|}\int_\Omega z(x)\ \mathrm{d}x \quad \text{for} \quad z\in H^1(\Omega)'\cap L^1(\Omega).$$

Next, for $z\in H^1(\Omega)'$ with $\langle z\rangle=0$, let $\mathcal{K}[z]\in H^1(\Omega)$ be the unique (variational) solution to

$$-\Delta\mathcal{K}[z] = z \text{ in } \Omega, \qquad \nabla\mathcal{K}[z]\cdot\mathbf{n}=0 \text{ on } \partial\Omega, \tag{1.4a}$$

satisfying

$$\langle\mathcal{K}[z]\rangle = 0. \tag{1.4b}$$

Also, for $p\in[1,\infty]$, we denote the positive cone of $L^p(\Omega)$ by $L^p_+(\Omega)$.

THEOREM 1.1. *Assume that γ satisfies (1.3) and consider $u^{in}\in L^1_+(\Omega)\cap H^1(\Omega)'$ and $v^{in}\in L^\infty_+(\Omega)\cap H^1(\Omega)$ with $M:=\langle u^{in}\rangle>0$. If (u,v) is a global weak solution to (1.1) in the sense of Definition 2.1 below, then*

$$\lim_{t\to\infty}\|\nabla P(t)\|_2 = \lim_{t\to\infty}\|v(t)\|_{H^1} = 0, \tag{1.5}$$

$$\lim_{t\to\infty}\int_t^{t+1}\|u(s)-M\|_2^2\ \mathrm{d}s = \lim_{t\to\infty}\int_t^{t+1}\|v(s)\|_{H^2}^2\ \mathrm{d}s = 0, \tag{1.6}$$

where $P(t):=\mathcal{K}[u(t)-M]$ for $t\ge 0$.

As already mentioned, Theorem 1.1 supplements previous results in the literature showing the long term convergence of $(u-M,v)$ to zero, either in low space dimension $n\in\{1,2,3\}$, see [8, Theorem 1.2], or when $\|v^{in}\|_\infty$ is sufficiently small, see [6, Theorem 1.2]. As in [8], the proof of Theorem 1.1 relies on the so-called duality estimate derived from (1.1a) (Proposition 2.1) and the dissipativity properties of (1.1b) (Lemma 2.2). The building block of the proof is to show that $\|\nabla P\|_2^2 + a\|v\|_2^2$ is a Liapunov functional for (1.1) for a suitable choice of $a>0$. This step is the main difference with the approach developed in [8] where a functional of the form $\|\nabla P\|_2^2+b\|\nabla v\|_2^2$ with $b>0$ is used.

REMARK 1.1. When $\gamma(0)=0$, Theorem 1.1 is no longer true and convergence of $u(t)$ as $t\to\infty$ to a non-constant limit may take place, see [11, Theorem 1.5] and [5].

We do not address here the issue of the existence of global solutions to (1.1) and refer to [6,7,11] for the existence of global bounded classical solutions and to [6–8,12,13] for that of global weak solutions under various assumptions on γ (with either $\gamma(0)=0$ or $\gamma(0)>0$) and the space dimension n. In particular, given a global weak solution (u,v) to (1.1) constructed in [7,8] and $t_0>0$, $(t,x)\mapsto(u,v)(t+t_0,x)$ is a weak solution to (1.1) in the sense of Definition 2.1, so that the convergence stated in Theorem 1.1 applies to these solutions.

We next combine (1.6), the time monotonicity of the L^∞-norm of v, and the regularizing effects of the heat semigroup to supplement the convergence (1.5) of v in $H^1(\Omega)$ with convergence to zero of v in $L^\infty(\Omega)$, thereby recovering the outcome of [8, Theorem 1.2].

Corollary 1.1. *Under the assumptions of Theorem 1.1, one has also*

$$\lim_{t\to\infty}\|v(t)\|_\infty=0.$$

2. Proofs

We begin with the definition of a global weak solution to (1.1) and introduce the Hilbert space

$$H^2_N(\Omega):=\{z\in H^2(\Omega)\ :\ \nabla z\cdot\mathbf{n}=0\ \text{ on }\ \partial\Omega\},$$

which is actually the domain of the Laplace operator in $L^2(\Omega)$ supplemented with homogeneous Neumann boundary conditions.

Definition 2.1. *Consider $u^{in}\in L^1_+(\Omega)\cap H^1(\Omega)'$ and $v^{in}\in L^\infty_+(\Omega)\cap H^1(\Omega)$. A global weak solution to (1.1) is a couple of non-negative functions*

$$(u,v)\in C_w([0,\infty),H^1(\Omega)')\times C([0,\infty),L^2(\Omega))$$

satisfying, for any $t>0$,

$$\begin{aligned}&u\in L^2((0,t)\times\Omega),\\ &v\in L^\infty((0,t)\times\Omega)\cap W^{1,2}((0,t),L^2(\Omega))\cap L^2((0,t),H^2_N(\Omega)),\end{aligned}$$

along with

$$\int_\Omega u(t)\vartheta(t)\ \mathrm{d}x-\int_\Omega u^{in}\vartheta(0)\ \mathrm{d}x=\int_0^t\int_\Omega\big[u\gamma(v)\Delta\vartheta+u\partial_t\vartheta\big]\ \mathrm{d}x\mathrm{d}s \tag{2.1a}$$

for all $\vartheta\in W^{1,2}((0,t),L^2(\Omega))\cap L^2((0,t),H^2_N(\Omega))$ and

$$\partial_t v-\Delta v+uv=0\ \ \text{a.e. in}\ \ (0,t)\times\Omega. \tag{2.1b}$$

We recall that, given a Banach space X and $T\in(0,\infty]$, $C_w([0,T),X)$ denotes the space of weakly continuous functions from $[0,T)$ to X.

We next derive several estimates on u and v which are already well-known, see [8]. From now on, $(c_i)_{i\ge1}$ denote positive constants depending only on Ω, γ in (1.3), u^{in}, and v^{in}.

Lemma 2.1. *For $t\ge0$,*

$$\langle u(t)\rangle=M=\langle u^{in}\rangle\ \ \text{and}\ \ \|v(t)\|_\infty\le V:=\|v^{in}\|_\infty. \tag{2.2}$$

Proof. Lemma 2.1 readily follows from (2.1a) (with $\vartheta\equiv1$), along with (2.1b), the non-negativity of uv, and the comparison principle. ∎

We next exploit the specific form of (1.1a) to derive some consequences of a so-called duality estimate on u. As a preliminary step, we observe that the continuity and positivity (1.3) of γ and the boundedness (2.2) of v imply that

$$\gamma_* := \min_{s\in[0,V]}\{\gamma(s)\} > 0. \tag{2.3}$$

We also recall the Poincaré-Wirtinger inequality: there is $c_1>0$ such that

$$\left\|z-\langle z\rangle\right\|_2 \le c_1\|\nabla z\|_2, \qquad z\in H^1(\Omega). \tag{2.4}$$

PROPOSITION 2.1. *Set $P=\mathcal{K}[u-M]$. Then, for $t>0$,*

$$P\in W^{1,2}((0,t),L^2(\Omega))\cap L^2((0,t),H^2_N(\Omega))$$

and

$$\|\nabla P(t)\|_2^2+\gamma_*\int_0^t \|u(s)-M\|_2^2 \,\mathrm{d}s \le \|\nabla P^{in}\|_2^2+c_2\int_0^t \|\nabla v(s)\|_2^2 \,\mathrm{d}s, \tag{2.5a}$$

$$\|\nabla P(t)\|_2^2 \le e^{-c_3t}\|\nabla P^{in}\|_2^2+c_2\int_0^t e^{c_3(s-t)}\|\nabla v(s)\|_2^2 \,\mathrm{d}s, \tag{2.5b}$$

with $P^{in}:=\mathcal{K}[u^{in}-M]$, $c_2:=\left[Mc_1\|\gamma'\|_{L^\infty(0,V)}\right]^2/\gamma_*$, and $c_3:=\gamma_*/c_1^2$.

Proof. As

$$\partial_t P = \langle u\gamma(v)\rangle - u\gamma(v) \ \text{ in } \ (0,\infty)\times\Omega \tag{2.6}$$

by (1.4a) and (2.1a) (with a suitable choice of test functions), the claimed regularity of P is a consequence of (1.3), (1.4), the square integrability of u, and the boundedness of v. It then follows from (2.6) that

$$\begin{aligned}\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\nabla P\|_2^2 &= -\int_\Omega \partial_t P\Delta P \,\mathrm{d}x = \int_\Omega (u-M)\left[\langle u\gamma(v)\rangle - u\gamma(v)\right] \,\mathrm{d}x \\ &= -\int_\Omega \gamma(v)u(u-M)\,\mathrm{d}x \\ &= -\int_\Omega \gamma(v)(u-M)^2\,\mathrm{d}x - M\int_\Omega \gamma(v)(u-M)\,\mathrm{d}x. \end{aligned} \tag{2.7}$$

We next use (1.3), (2.2), (2.4), and Hölder's inequality to estimate the second term on the right-hand side of (2.7) and obtain

$$\begin{aligned}\left|M\int_\Omega \gamma(v)(u-M)\,\mathrm{d}x\right| &= M\left|\int_\Omega \left[\gamma(v)-\gamma(\langle v\rangle)\right](u-M)\,\mathrm{d}x\right| \\ &\le M\|\gamma'\|_{L^\infty(0,V)}\int_\Omega |v-\langle v\rangle||u-M|\,\mathrm{d}x \\ &\le M\|\gamma'\|_{L^\infty(0,V)}\|v-\langle v\rangle\|_2\|u-M\|_2 \\ &\le Mc_1\|\gamma'\|_{L^\infty(0,V)}\|\nabla v\|_2\|u-M\|_2. \end{aligned} \tag{2.8}$$

We therefore infer from (2.3), (2.7), (2.8), and Young's inequality that

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\nabla P\|_2^2 \le -\gamma_*\|u-M\|_2^2+\frac{\gamma_*}{2}\|u-M\|_2^2+\frac{c_2}{2}\|\nabla v\|_2^2,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\|\nabla P\|_2^2+\gamma_*\|u-M\|_2^2\le c_2\|\nabla v\|_2^2, \tag{2.9}$$

and obtain (2.5a) after time integration of (2.9). Finally, by (1.4), (2.4), and Hölder's inequality,

$$\begin{aligned}\|\nabla P\|_2^2&=-\int_\Omega P\Delta P\ \mathrm{d}x=\int_\Omega (u-M)P\ \mathrm{d}x\\&\le\|u-M\|_2\|P\|_2\le c_1\|u-M\|_2\|\nabla P\|_2,\end{aligned}$$

so that

$$\|\nabla P\|_2\le c_1\|u-M\|_2.$$

Combining (2.9) and the above inequality, we find

$$\frac{\mathrm{d}}{\mathrm{d}t}\|\nabla P\|_2^2+c_3\|\nabla P\|_2^2\le c_2\|\nabla v\|_2^2,$$

from which (2.5b) follows after time integration. ☐

We next take advantage of the non-positivity of the right-hand side of (1.1b) to obtain a classical energy estimate on v.

LEMMA 2.2. *For $t\ge 0$,*

$$\frac{\mathrm{d}}{\mathrm{d}t}\|v\|_2^2+2\|\nabla v\|_2^2+2\|v\sqrt{u}\|_2^2=0.$$

At this point, we deviate from the proof of [8, Theorem 1.2] and construct a Liapunov functional associated with (1.1), building upon the outcome of Proposition 2.1 and Lemma 2.2. This is clearly the main building block of the proof.

PROPOSITION 2.2. *For $t\ge 0$,*

$$\|\nabla P(t)\|_2^2+c_2\|v(t)\|_2^2+\int_0^t\left[\gamma_*\|u(s)-M\|_2^2+c_2\|\nabla v(s)\|_2^2\right]\ \mathrm{d}s\le\|\nabla P^{in}\|_2^2+c_2\|v^{in}\|_2^2.$$

Proof. Let $t>0$. We infer from (2.3), (2.5a), (2.8), Lemma 2.2, and Young's inequality that

$$\begin{aligned}&\|\nabla P(t)\|_2^2+\gamma_*\int_0^t\|u(s)-M\|_2^2\ \mathrm{d}s+c_2\|v(t)\|_2^2+2c_2\int_0^t\|\nabla v(s)\|_2^2\ \mathrm{d}s\\&\le\|\nabla P^{in}\|_2^2+c_2\int_0^t\|\nabla v(s)\|_2^2\ \mathrm{d}s+c_2\|v^{in}\|_2^2,\end{aligned}$$

from which Proposition 2.2 readily follows. ☐

We next argue as in [8, Lemma 3.2] to obtain additional information on v.

LEMMA 2.3. *For $t\ge 0$,*

$$\|\nabla v(t)\|_2^2\le e^{-2Mt}\|\nabla v^{in}\|_2^2+V^2\int_0^t e^{2M(s-t)}\|u(s)-M\|_2^2\ \mathrm{d}s, \tag{2.10a}$$

$$\int_0^t\|\Delta v(s)\|_2^2\ \mathrm{d}s\le\|\nabla v^{in}\|_2^2+V^2\int_0^t\|u(s)-M\|_2^2\ \mathrm{d}s. \tag{2.10b}$$

Proof. We infer from (2.1b), (2.2), and Hölder's and Young's inequalities that

$$\begin{aligned}\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|\nabla v\|_2^2 &= -\int_\Omega \partial_t v\Delta v\ \mathrm{d}x = -\|\Delta v\|_2^2 + \int_\Omega uv\Delta v\ \mathrm{d}x \\ &= -\|\Delta v\|_2^2 + \int_\Omega (u-M)v\Delta v\ \mathrm{d}x + M\int_\Omega v\Delta v\ \mathrm{d}x \\ &\le -\|\Delta v\|_2^2 + V\|u-M\|_2\|\Delta v\|_2 - M\|\nabla v\|_2^2 \\ &\le -\frac{\|\Delta v\|_2^2}{2} + \frac{V^2\|u-M\|_2^2}{2} - M\|\nabla v\|_2^2.\end{aligned}$$

Hence,

$$\frac{\mathrm{d}}{\mathrm{d}t}\|\nabla v\|_2^2 + 2M\|\nabla v\|_2^2 + \|\Delta v\|_2^2 \le V^2\|u-M\|_2^2,$$

from which Lemma 2.3 follows. ∎

Summarizing the outcome of Proposition 2.2 and Lemma 2.3, we have so far obtained the following estimates on u and v.

PROPOSITION 2.3. *There is $c_4>0$ such that*

$$\|P(t)\|_{H^1} + \|v(t)\|_{H^1} \le c_4, \qquad t\ge 0, \tag{2.11}$$

$$\int_0^\infty \left[\|u(s)-M\|_2^2 + \|\nabla v(s)\|_2^2 + \|\Delta v(s)\|_2^2\right]\ \mathrm{d}s \le c_4, \tag{2.12}$$

and

$$\lim_{t\to\infty}\|\nabla P(t)\|_2 = \lim_{t\to\infty}\|\nabla v(t)\|_2 = 0. \tag{2.13}$$

Proof. The bounds (2.11) and (2.12) being immediate consequences of (1.4b), (2.4), Proposition 2.2 and Lemma 2.3, we are left with proving (2.13). To this end, we recall that, if F belongs to $L^1(0,\infty)$, then a straightforward consequence of the Lebesgue dominated convergence theorem is that

$$\lim_{t\to\infty}\int_0^t e^{\alpha(s-t)}|F(s)|\ \mathrm{d}s = 0 \tag{2.14}$$

for any $\alpha>0$. Thanks to (2.12), $s\mapsto\|\nabla v(s)\|_2^2$ and $s\mapsto\|u(s)-M\|_2^2$ both belong to $L^1(0,\infty)$ and we use (2.14) (first, with $\alpha=c_3$ and $F=\|\nabla v\|_2^2$, and then with $\alpha=2M$ and $F=\|u-M\|_2^2$) to take the limit $t\to\infty$ in (2.5b) and (2.10a) and obtain (2.13), thereby completing the proof. ∎

The final step of the proof of Theorem 1.1 deals with the convergence of $\|v(t)\|_1$ as $t\to\infty$.

LEMMA 2.4.

$$\lim_{t\to\infty}\|v(t)\|_1 = 0.$$

Proof. We infer from (2.1b), (2.2), and the non-negativity of v that, for $t\ge 0$,

$$\frac{\mathrm{d}}{\mathrm{d}t}\|v\|_1 = -\int_\Omega uv\ \mathrm{d}x = -\int_\Omega (u-M)v\ \mathrm{d}x - M\|v\|_1 = -\int_\Omega (u-M)(v-\langle v\rangle)\ \mathrm{d}x - M\|v\|_1.$$

Hence, by (2.4) and Hölder's inequality,

$$\frac{\mathrm{d}}{\mathrm{d}t}\|v\|_1 + M\|v\|_1 \le \|u-M\|_2\|v-\langle v\rangle\|_2 \le c_1\|u-M\|_2\|\nabla v\|_2.$$

We then integrate with respect to time to find

$$\|v(t)\|_1 \le e^{-Mt}\|v^{in}\|_1 + c_1\int_0^t e^{M(s-t)}\|u(s)-M\|_2\|\nabla v(s)\|_2 \ \mathrm{d}s. \tag{2.15}$$

Since $s \mapsto \|u(s)-M\|_2\|\nabla v(s)\|_2$ belongs to $L^1(0,\infty)$ by (2.12), we deduce from (2.14) (with $\alpha = M$ and $F = \|u-M\|_2\|\nabla v\|_2$) that the right-hand side of (2.15) converges to zero as $t \to \infty$. Consequently,

$$\lim_{t\to\infty}\|v(t)\|_1 = 0,$$

and the proof is complete. ◻

Theorem 1.1 is now an immediate consequence of Proposition 2.3 and Lemma 2.4.

Proof. (**Proof of Theorem 1.1.**) The convergences (1.5) follow from (2.4), (2.13), and Lemma 2.4, while the time integrability (2.12) of $\|u-M\|_2$ and $\|\Delta v\|_2$, along with (1.5) and elliptic regularity, gives (1.6). ◻

We finally provide the proof of Corollary 1.1.

Proof. (**Proof of Corollary 1.1.**) Let $t>0$. Since v is a non-negative strong solution to (2.1b), the Duhamel formula gives

$$0 \le v(t+\tau) = e^{\tau\Delta}v(t) - \int_0^\tau e^{(\tau-s)\Delta}(uv)(s+t) \ \mathrm{d}s, \qquad \tau \ge 0,$$

where $\left(e^{\tau\Delta}\right)_{\tau\ge 0}$ denotes the semigroup generated in $L^2(\Omega)$ by the Laplace operator with domain $H^2_N(\Omega)$ (corresponding to homogeneous Neumann boundary conditions). Owing to the non-negativity of u, v, and $\left(e^{\tau\Delta}\right)_{\tau\ge 0}$, we further obtain

$$0 \le v(t+\tau) \le e^{\tau\Delta}v(t), \qquad \tau \ge 0.$$

Consequently, for $\tau \ge 0$,

$$\|v(t+\tau)\|_\infty \le \left\|e^{\tau\Delta}v(t)\right\|_\infty,$$

and we infer from the regularizing properties of the heat semigroup [10, Lemma 3, p. 25] that

$$\|v(t+\tau)\|_\infty \le c_5 \min\{1,\tau\}^{-n/2}\|v(t)\|_1.$$

Combining (1.5) and the above inequality (with $\tau = 1$) leads us to

$$\lim_{t\to\infty}\|v(t+1)\|_\infty \le \lim_{t\to\infty}\|v(t)\|_1 = 0,$$

and completes the proof. ◻

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