Homology, Homotopy and Applications, vol. 25(2), 2023, pp.403–405

ERRATUM TO "FROM LOOP GROUPS TO 2-GROUPS"

JOHN C. BAEZ, ALISSA S. CRANS, URS SCHREIBER AND DANNY STEVENSON

(*communicated by J. Daniel Christensen*)

Abstract

There were a number of sign errors in our paper "From loop groups to 2-groups" [*Homology Homotopy Appl.* **9** (2007), 101–135]. Here we explain how to correct those errors.

The following sign corrections to our paper [**BCSS**] make the article consistent with [**BC**] and [**LM**], in particular, by correcting our definition of morphisms between 2-term L_∞-algebras. The definition given in [**BC**], Definition 4.3.4, was independently checked by Kevin van Helden to be consistent with the one suggested in [**LM**, Remark 5.3]. See also [**H**, Definition 2.5]. Since the main theorem of the paper involves several such morphisms, a number of signs in the data of some L_∞-algebra morphisms need to be changed. Moreover, the definition of the crossed module action α in Proposition 2.4 led to an inconsistency independent of the L_∞-algebra material, found by David Michael Roberts. The corrected sign is self-consistent, as well as agreeing with the rest of the paper. There were additionally some typos in an earlier paper [**MS**] leading to some innocuous sign errors that do not impact the other calculations.

All the corrections here have been made in the current arXiv version of our paper [**BCSS**]. All of our calculations have been independently checked by Rist, Saemann and Wolf [**RSW**], as well as by Roberts.

- The big commutative diagram in Definition 2.2 should be replaced by

$$\begin{tikzcd}
[[F_0(x), F_0(y)], F_0(z)] \arrow[r, "J_{F_0(x),F_0(y),F_0(z)}"] \arrow[d, "{[F_2,1]}"'] & {[F_0(x), [F_0(y), F_0(z)]] + [[F_0(x), F_0(z)], F_0(y)]} \arrow[d, "{[1,F_2]+[F_2,1]}"] \\
{[F_0[x,y], F_0(z)]} \arrow[d, "F_2"'] & {[F_0(x), F_0[y,z]] + [F_0[x,z], F_0(y)]} \arrow[d, "F_2+F_2"] \\
F_0[[x,y],z] \arrow[r, "F(J_{x,y,z})"] & F_0[x,[y,z]] + F_0[[x,z],y]\,.
\end{tikzcd}$$

Received August 9, 2023; published on November 22, 2023.
2020 Mathematics Subject Classification: 18G45, 22E67, 53C08
Key words and phrases: L_∞-algebra, loop group, 2-group
Article available at `http://dx.doi.org/10.4310/HHA.2023.v25.n2.a18`

- Equation (5) needs to be replaced by

$$\begin{aligned}\phi_1\big(l_3(x,y,z)\big) - l_3\big(\phi_0(x),\phi_0(y),\phi_0(z)\big) = \\ \phi_2\big(x,l_2(y,z)\big)+\phi_2\big(y,l_2(z,x)\big)+\phi_2\big(z,l_2(x,y)\big)+ \\ l_2\big(\phi_0(x),\phi_2(y,z)\big)+l_2\big(\phi_0(y),\phi_2(z,x)\big)+l_2\big(\phi_0(z),\phi_2(x,y)\big)\,.\end{aligned}$$

- The definition of κ in equation (12) should be

$$\kappa(f,g) = \exp\left(-2ik\int_0^{2\pi}\int_0^{2\pi}\langle f(t)^{-1}f'(t), g'(\theta)g(\theta)^{-1}\rangle\, d\theta\, dt\right),$$

correcting a typo in [**MS**].

- The definition of the normal subgroup N below equation (12) should read

  "Let N be the subset of $P_0\Omega G\times U(1)$ consisting of pairs (γ,z) such that $\gamma\colon [0,2\pi]\to\Omega G$ is a loop based at $1\in\Omega G$ and

$$z = \exp\left(ik\int_{D_\gamma}\omega\right),$$

  where D_γ is any disk in ΩG with γ as its boundary."

  for consistency with the definition of κ above.

- The definition of $d\alpha$ in the statement of Proposition 3.1 should be

$$d\alpha(p)(\ell,c) = \left([p,\ell],\ 2k\int_0^{2\pi}\langle\ell(\theta),p'(\theta)\rangle\, d\theta\right).$$

- The definition of β_p in the proof of Proposition 3.1 should be

$$\beta_p(\xi) = 2\int_0^{2\pi}\langle\xi(\theta),p(\theta)^{-1}p'(\theta)\rangle\, d\theta\,.$$

- The definition of ϕ_2 in the statement of Lemma 5.4 should be

$$\phi_2(p_1,p_2) = k\int_0^{2\pi}\Big(\langle p_2,p_1'\rangle - \langle p_2',p_1\rangle\Big)\, d\theta\,.$$

- The definition of λ_2 in the proof of Lemma 5.5 should be

$$\lambda_2(\ell_1,\ell_2) = \left(0, 2k\int_0^{2\pi}\langle\ell_1,\ell_2'\rangle\, d\theta\right).$$

With these changes, the calculations in the proofs all go through, leaving the results unchanged.

References

[BC] J. Baez and A. S. Crans, Higher-dimensional algebra VI: Lie 2-algebras, *Theor. Appl. Categ.* **12** (2004), 492–528. Also available as arXiv:math/0307263.

[BCSS] J. C. Baez, A. S. Crans, U. Schreiber and D. Stevenson, From loop groups to 2-groups, *Homology Homotopy Appl.* **9** (2007), 101–135. Corrected version available as arXiv:math/0504123v3.

[H] K. S. van Helden, Classification of 2-term L_∞-algebras, Available as arXiv:2109.10202.

[LM] T. Lada and M. Markl, Strongly homotopy Lie algebras, *Commun. Alg.* **6** (1995), 2147–2161. Also available as arXiv:hep-th/9406095.

[MS] M. K. Murray and D. Stevenson, Higgs fields, bundle gerbes and string structures, *Commun. Math. Phys.* **243** (2003), 541–555. Also available as arXiv:math/0106179.

[RSW] D. Rist, C. Saemann and M. Wolf, Explicit non-abelian gerbes with connections. Available as arXiv:2203.00092.

John C. Baez baez@math.ucr.edu

Department of Mathematics, University of California at Riverside, USA

Alissa S. Crans acrans@lmu.edu

Department of Mathematics, Loyola Marymount University, USA

Urs Schreiber urs.schreiber@googlemail.com

Mathematics Division, New York University in Abu Dhabi, UAE

Danny Stevenson daniel.stevenson@adelaide.edu.au

School of Computer and Mathematical Sciences, The University of Adelaide, Australia