Fields Medalists of ICM 2018

Note from the editor: The Fields Medals of ICM 2018 were awarded to Caucher Birkar (Cambridge University), Alessio Figalli (ETH Zurich), Peter Scholze (University of Bonn), and Akshay Venkatesh (Stanford University). The following article is an excerpt of the news released by IMU.

Caucher Birkar

Caucher Birkar has made fundamental contributions to birational geometry in two particular areas: the minimal model program (MMP) and the boundedness of Fano varieties. The original MMP involves two kinds of projective varieties Y with so-called terminal singularities whose canonical divisors K have opposite properties: for a minimal model K is nonnegative on curves on Y; while for a Fano fibering Y has a surjective morphism onto a lower-dimensional projective variety with K relatively ample. The MMP attempts to construct for each smooth projective variety a birational map to either a minimal model or a Fano fibering.

Although the MMP is not always known to work, Birkar jointly with Cascini, Hacon, and McKernan made a stunning contribution; a special version of the MMP works for complex varieties of arbitrary dimension whose canonical divisor is either big or not pseudo effective, a situation which covers many important cases. They actually established the MMP for a wider class of singularities, which was essential for the induction on dimension in the proof, and it implies many important consequences such as the finite generation of canonical rings of arbitrary smooth projective varieties. The MMP is now a fundamental tool which is used extensively.

It was Birkar who further proved that complex Fano varieties (i.e., Fano fiberings over a point) of arbitrary fixed dimension with terminal singularities are parametrized by a (possibly reducible) algebraic variety. Since these Fano varieties constitute one of the main outputs of MMP as applied to smooth projective varieties, their boundedness, previously considered unreachable, is fundamentally important. Birkar has settled the more general Borisov–Alexeev–Borisov conjecture building upon results by Hacon, McKernan, Xu, and others. Birkar's boundedness will be crucial as a paradigm for the full MMP.

Alessio Figalli

Alessio Figalli has made multiple fundamental advances in the theory of optimal transport, while also applying this theory in novel ways to other areas of mathematics. Only a few of his numerous results in these areas are described here.

Figalli's joint work with De Philippis on regularity for the Monge–Ampère equation is a groundbreaking result filling the gap between gradient estimates discovered by Caffarelli and full Sobolev regularity of the second derivatives of the convex solution of the Monge–Ampère equation with merely bounded right-hand side. The result is almost optimal in view of existing counterexamples. It has direct implications on regularity of the optimal transport maps, and on regularity to semigeostrophic equations.

Figalli initiated the study of the singular set of optimal transport maps and obtained the first definite results in this direction: he showed that it has null Lebesgue measure in full generality. He has also given significant contributions to the theory of obstacles problems, introducing new methods to analyze the structure of the free boundary.

Figalli and his coauthors have also applied optimal transport methods in a striking fashion to obtain sharp quantitative stability results for several fundamental geometric inequalities, such as the isoperimetric and Brunn–Minkowski inequalities, without any additional assumptions of regularity on the objects to which these inequalities are applied; the methods are also not reliant on Euclidean symmetries, extending in particular to the Wulff inequality to yield a quantitative description of the low-energy states of crystals.

Peter Scholze

Peter Scholze has transformed arithmetic algebraic geometry over p-adic fields.

Scholze's theory of perfectoid spaces has profoundly altered the subject of p-adic geometry by relating it to geometry in characteristic p. Making use of this theory, Peter Scholze proved Deligne's weight-monodromy conjecture for complete intersections. As a further application, he constructed Galois representations that are attached to torsion cohomology classes of locally symmetric spaces, resolving a long-standing conjecture.

Scholze's version of p-adic Hodge theory extends to general p-adic rigid spaces. Together with Bhatt and Morrow, Scholze developed an integral version of p-adic Hodge theory that establishes a relation between the torsion in Betti and crystalline cohomologies.

On the way to the revolution that he launched in arithmetic geometry, Scholze took up a variety of topics that he reshaped, such as algebraic topology and topological Hochschild homology.

Scholze developed new cohomological methods. Beyond p-adic fields, Scholze's vision of a cohomology theory over the integers has become a guideline that fascinates the entire mathematical community.

Akshay Venkatesh

Akshay Venkatesh has made profound contributions to an exceptionally broad range of subjects in mathematics, including number theory, homogeneous dynamics, representation theory and arithmetic geometry. He solved many longstanding problems by combining methods from seemingly unrelated areas, presented novel viewpoints on classical problems, and produced strikingly far-reaching conjectures.

What follows is a small sample of his major achievements:

Venkatesh introduced a general and unifying technique based on representation theory and homogeneous dynamics in the subconvexity problem for L-functions and (partly in collaboration with Michel) used these ideas to give a complete treatment of all cases of subconvexity for $GL(2)$ over number fields.

He made major progress on the local-global principle for the representations of one quadratic lattice by another, in joint work with Ellenberg.

In joint work with Einsiedler, Lindenstrauss and Michel, Venkatesh proved equidistribution of the periodic torus orbits in $SL(3,Z)\backslash SL(3,R)$ that are attached to the ideal classes of totally real cubic number fields as the discriminant tends to infinity.

Venkatesh established effective equidistribution of periodic orbits of many semisimple groups both in the local and adelic settings, in joint work with Einsiedler, Margulis, and in part with Mohammadi.

With Ellenberg and Westerland, Venkatesh established significant special cases of the Cohen–Lenstra conjectures concerning class groups in the function field setting.