From fields to Fields

by Caucher Birkar*

This short text is the story of my journey to become a mathematician. This is by no means a complete biography. Even my mathematical research will be discussed briefly. Apologies in advance to the many friends whose names are not mentioned here. Thanks to anyone who has helped me in my life in any way.

The title "From fields to Fields" is a shorter version of a similar title of an article published by the University of Nottingham. I picked the title for a few reasons. First, it refers to my childhood when me and my family worked on farming fields moving from one field to another. Second, it refers to the journey from farming fields to the Fields Medal. Third, it refers to the notion of field which is central in many parts of modern mathematics especially algebraic geometry in which one chooses a ground field from the beginning and then changing the field can reveal interesting properties often of an arithmetic nature.

Figure 1. Ne Village, with family, around 1984.

Family Background

I was born in Summer 1978 in the Ne village near the city of Marivan, Kurdistan, Iran on the Eastern side of the border between Iran and Iraq. I was the third child after an older brother and an older sister. In later years three younger brothers joined. I lived in Ne until I was 18. This is where I learnt mathematics and fell in love with it.

My ancestors were aristocrats owning land in the area. However, sometime before I was born the land was redistributed by the government so we lost the privileges my ancestors enjoyed. My parents were farmers owning small pieces of land just as did many other families. My father, Majid, had attended primary school but my mother, Sakina, did not. My grandfather could read and write who learnt it in the traditional education system before modern schools came to the village. My uncles attended school long before I did.

Farming was the main source of income for people living in villages. People grew all sorts of crops from numerous kinds of fruits and vegetables to wheat, barley, corn, etc. Much of the food grown was for home consumption so people did not make much money out of selling the surplus. Thus one can say that people were not poor yet they had little money as they were self-sufficient to a good extent.

Many families had several small pieces of farming land usually far from each other. Each morning they would walk or ride an animal or a tractor to one of these pieces and worked there until late afternoon. Afterwards they would return to their houses in the village.

Being part of a farming community meant that children started to work on the farm from a young

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age, first by helping with light tasks and gradually with more heavy work. I was no exception to this rule. I and my siblings worked on the farm from early Spring to early Autumn after school hours, sometime even during the final examination period in late Spring.

Farming was only partially mechanised so we had to do lots of physically tiring work. On the plus side, people were mentally happy as this job did not put them under psychological stress unlike many other kind of jobs in the modern world.

When I was in high school and immersed in the world of mathematics, I often thought about mathematics problems while farming. When I had some ideas I would write them down on a small notebook that I carried with me. For example, I remember trying to generalise to higher dimension a three-dimensional theorem in projective geometry while riding a tractor on my way to a farm.

Not everyone in the village did farming. At some point, the majority of people in the village were not original inhabitants but rather immigrants from the city or other villages close to the border who sought refuge in our village due to the ongoing Iran-Iraq war. Those from the city worked as civil servants in public organisations or owned small shops. Many of the immigrants from other villages continued to stay after the war.

War

When I was born Iran was already unstable. A few months later the revolution overthrew the government. Two years after, the Iran-Iraq war started. All these made life difficult especially for us living few kilometres away from the border. The war lasted eight years until when I was in fifth grade in primary school. A large number of people died in my region due to frequent aerial bombing including chemical bombs in some instances. I remember seeing rows of bombs shining in the blue sky falling on the city. We in the village would immediately run to the nearby mountains bordering the village when we heard the aeroplanes roaring. In my case I would run to my grandfather's house which was on the side of the mountain.

Due to the mountains shielding us, my village was generally safer than the city and many other villages closer to the border. Thus a large number of people migrated to live in the village probably tripling the population. Some of those left after the war but many continued to stay. However, sometimes my village was also not safe enough so we would have to migrate to other far away villages.

Apart from the bombing there were other hazards related to the war. For example, explosives were abundant. We children sometimes played with these, even put them in fire for the fun of it. Although amazingly none of my friends got hurt in such games but there were casualties among other groups of children – years later three children found an unexploded bomb from the war era which exploded and instantly killed them. My uncle died only a few years ago while working on a field and accidentally hitting an unexploded bomb by his shovel.

War disrupts everything including education and I am grateful to the teachers who taught us patiently during those difficult years. The other side of the border of my home town were also Kurds but living in Iraq. We shared the same culture, the same language, and had family connections.

Primary School

The first time I got a taste of school was when I was five years old living temporarily in the Shekha-Kwera village far from my village. My father used to take me to the village school and sat with me in the class so that I get used to school. I was not formally registered in the school since I was less than six. It was a very useful experience for me. At the time there was no preschool education available in the region, at least not in villages, so some students were very stressful when they entered school for the first time.

After a few months we returned to our village. In the Autumn 1984 I started formal education. Our school was an ordinary old house with three rooms. Our class was very simple with benches, desks, a blackboard, and a stove for heating. Our teacher was a women as were my teachers in grades two and three. Boys and girls were mixed but were separated from grade four.

Due to my father's emphasis on education, both I and my older brother Haidar were doing well in school. In grade two the teacher announced one day that I was number one in the class. In grade three however something funny happened. We were expecting examination results. There was a rumour that my cousin who was my classmate had failed mathematics. After school I told my Aunt that my cousin has failed in mathematics. A few days later it turned out that she had passed but I had failed!

One of the really bad common practices of education at the time was physical beating usually with a stick or wires. Students were beaten either because they could not answer some questions or for misbehaving or arriving late. I probably had more than my fair share of beating. This continued until end of middle school. Some of the teachers were more cruel, beating students possibly due to their own personal problems. The worst experience I had was when I was in grade four. Our teacher, a man from a far away

city, would beat us so bad with a bundle of wires. I remember little from that year apart from the beating. In other instances, some teachers forced students to hold their hands in the snow after which they would be beaten with a stick.

Fortunately in grade five our teacher, Rahman Konaposhi, was really good. He did not even have a diploma but was better than many of the other "qualified" teachers. He was a peaceful and calm man so beating was not his way of teaching. He used novel techniques to teach. For instance, every day he told us a nice story before we left school. In Spring he used to take us out and taught us in the beautiful nature nearby school. He also organised competitions with schools in nearby villages by meeting somewhere halfway between the villages. He even arranged for a blackboard to be transported to the site on the back of a donkey.

It was that year that I felt that I had a connection with mathematics. I got good results in the mathematics exams early in the year although I did not spend much time preparing. That year my friend, Kamaran Kasraei, and I were top students in the class. Our rivalry lasted for few years but we have continued to be close friends ever since.

My brother Haidar, six years older than me, had the biggest influence on my education. Having him around as a role model was a blessing. He was a truly unique person, very peaceful, pleasant, and creative. He was always busy inventing and building things. When I was five he used to make paper parachutes and even sold some of them to children. One of the things that impressed me was when he made a projector and a short animation film. He showed it to the children in the neighbourhood in a dark room. By carving out alphabet letters on a plastic sheet he could print texts similar to a type-machine. He built his own gym equipment including dumbbells and barbells using iron bars and concrete. He also told me and my younger brother his home-made stories some of which I recycled decades later telling them to my son. He was also excellent in calligraphy. Although we lived in the same house, often playing or studying in the same room but I remember only one instance that I had a confrontation with him, even that was my fault. He eventually entered university to study electronics engineering and after graduation he started teaching in a technical high school.

Middle School

I attended both primary and middle school in my village. Middle school consisted of three years. Haidar started to teach me mathematics and physics which was not covered in middle school but rather appeared in high school. For example, he taught me some basics of calculus such as differentiation and integration. He taught me such things for their beauty and excitement rather than getting good results in school.

It was during middle school that my creative side flourished. I spent hours and days thinking about building or inventing certain things, for example, a machine to help with work on the farm. Although most were not successful partly due to lack of access to the necessary material but I believe the important thing was the concentration and the ambition which proved to be extremely useful in later years.

Together with some friends we created a theatre group. We played for children in the village. We made little money out of it but thoroughly enjoyed the comedy.

Together with my friend Kamaran we built a kite using wood and plastic. We took it on top of a nearly mountain and hoped to fly with it. Unfortunately it did not work partly because of the weight of the material we used and partly we did not quite understand the aerodynamics involved. I got even more ambitious as I spent quite some time trying to build an aeroplane and fly with it which never materialised.

The rivalry with Kamaran during those years was at its peak. This forced both of us to work hard and be the best in school. Although I tried to get good marks in every subject but clearly I preferred mathematics and in general natural sciences.

High School

High school consisted of four years. There was no high school in our village so I had to commute to the nearby Marivan city which is only few kilometres away, except for three months that I spent at my Aunt's house in the city. Commuting in the cold Winter months was especially tough. In Spring, sometimes I would simply walk home from the city to save money or to enjoy the nature. It took me about 40 minutes to walk.

Apart from the commuting, studying in town was also initially though in the sense that I did not think that a villager could compete with all the excellent students in the city. In the first year, I worked hard to make sure that I did not fall behind. Gradually I learnt that the city students were not as scary as it seemed in the beginning so I grew more confident and focused more on following my interests. In the second year, a team consisting of myself and two other classmates won an academic competition among all the schools in the city. We then entered a subsequent competition for all the selected schools in the Kurdistan province and to my surprise we won the number one spot.

As before I had special interests in mathematics and physics. It was in the second year that my passion

for mathematics intensified and I became convinced that mathematics was also the reason I liked physics. With the experience from middle school of learning outside the school system I indulged myself in reading books from the city library. For some reason the library had excellent mathematics books although very few or none other than me had any interest in them.

Reading those books and spending lots of time thinking about mathematics was not about getting good grades. I think I liked mathematics for various reasons. I thought it was beautiful, precise, immortal, challenging, a source of joy, and useful. To put it simply mathematics was in harmony with my human nature.

Although I tried to learn many topics throughout high school but geometry somehow stood out. One of the topics I liked was projective geometry. For example, one can project a regular triangle onto any other triangle which can be used to prove may results about triangles. I liked the fact that a single idea could be applied to many different problems. I tried to generalise this to higher dimensions which I briefly mentioned above.

One of the books that I really enjoyed was the *Men of Mathematics* by E. T. Bell which consists of the biography of mathematicians from ancient world to early 20th century. I was really excited by the lives of these people and their discoveries.

Another great book was the *What is Mathematics?* by R. Courant and H. Robbins which introduces various areas of mathematics including algebra, topology, geometry, number theory, etc. This was really useful as the book goes well beyond high school mathematics textbooks while still keeping it relatively elementary. Yet another nice book was the *Mathematical Analysis* of T. M. Apostol.

Until I finished high school I left my home city twice only, on trips to the nearby provincial capital city of Sanandaj. The first trip was to participate int he mathematics competition mentioned above. On the second trip, my friend Kamran bought the analysis book of Apostol which then gave me a gift.

Influenced by earlier experiences I felt that learning mathematics alone is not enough. I thought that the ultimate satisfaction is in creating new mathematics. I was then determined to do research. I did not prove anything significant but the experience of trying to do research proved valuable in later years. It was around this time that I decided to become a professional mathematician. I was around 15–16 years old.

Although I enjoyed mathematics a lot but there was no one who could give me some advice, some direction. This total isolation was on one hand a blessing in disguise so that I could become more independent but on the other hand it was tough. It was hard to spend so much time on some thing that I could not discuss with others. I tried to get some of my friends to follow mathematics or any other science seriously but unfortunately it was fruitless. There were a number of really good students but their goal was to get a good degree in engineering or medicine and then land a good job.

In the final year of high school I decided to quit the standard school system and instead enrolled in a different system which had no classes and lectures but only examinations. This system was in place mainly for grown ups who had a full time job.

To enter university one had to finish high school and pass a national examination. It did not matter how good the high school results were. This was to the advantage of people living in less developed areas because the national examination was computerised so it was free of people's biases and prejudices. It made no difference which school one attended as long as one did well in this examination. Like many others I made good use of this system and selected only pure mathematics in several universities. I was picked for my second choice on the list, the University of Tehran. It was generally considered to be the second best university in the country after Sharif University of Technology.

The day the university entrance results were published in the newspapers was a big day for students. On that day, people would ask whether I had been accepted. On my way back home, at least two of my relatives told me "shame on you" when I told them that I was going to study pure mathematics.

Apart from mathematics I followed other interests in high school. For example, I used to write poetry. Influenced by Haidar I was also interested in electronics. Kamran and I spent months trying to build a short range FM radio broadcaster. We had the map of the circuit but the problem was finding the pieces, that is, transistors, resistors, capacitors, etc. There was no shop in the city to buy unused pieces. Instead we had to visit every electronic repair shop in the town who would tell us to look for these pieces among a pile of broken radios and other electronic equipment. That is why it took us a long time to find the pieces. We put them together according to the circuit map but first it did not work. On a second try it worked and we were very happy.

One of my aunts happened to be my neighbour. We told her to turn on her radio as there are some very important announcements to be made. We were secretly talking on the FM broadcaster and she was listening. We said that the world is going to end in a couple of days, everyone should prepare. Poor aunt took it serious and almost scared to death. We then told her that it was just a joke.

I had a bunch of close friends with whom I had lots of fun but they did not study that hard. As a result I had to stay awake the night before nearly every mathematics examination teaching them the mathematics they did not learn for months.

Undergraduate

In the Autumn 1996 I entered University of Tehran to study pure mathematics. Our class which included applied mathematics students consisted of 40 students half of which were female. Similar to when I entered high school I worried that I would not be able to catch up with all the students coming from much richer and more developed cities around the country. So in the first year I worked hard on following the lectures and passing examinations.

Before entering university most books that I read were written (or translated to) Persian. Kurdish was not taught in school so I and my friends learnt the language at home and from older people. English was taught in school but very few learnt enough to read books in this language. From the beginning of undergraduate I decided to read mathematics in English as there were a lot more advanced mathematics books available in English. The faculty had an excellent library.

Similar to high school I continued reading books not related to first year courses. Having learnt some basics of topology in high school I picked a book on algebraic topology and learnt more advanced topics such as fundamental groups and homology groups. I was fascinated by the use of algebra in classifying geometric spaces.

In the second year, I got interested in algebra especially commutative algebra. That year I met another Kurdish mathematics student, Reza Sazeedeh, who was doing a master degree. He was quite surprised by my knowledge of commutative algebra as it was his specialty subject. I tried to do some elementary research by investigating groups of homomorphisms between modules but apparently these were all known already.

Another topic that I followed in second year was mathematical logic, particularly universal algebra and model theory. A dream I had in this period was to somehow write two mathematical subjects in such a formal way that one could use a theorem in one subject, "translate" it and get a non-trivial result in the other subject for free. Due to lack of experience and ambition of the project I did not get anywhere.

In third year I happened to be room-mate with Reza. He spent quite sometime reading a photocopy of a book sometimes with excitement and sometimes with frustration. That book was the *Basic Algebraic Geometry* book of I. R. Shafarevich which is often used to teach a first graduate course on algebraic geometry. I slowly got interested in this book and started reading it. It was very exciting to see that many of the things that I had learnt in commutative algebra actually had a geometric meaning. I saw a dictionary between algebra and geometry which was much stronger than the connection I observed while learning algebraic topology. It was not long before I was convinced that this is the subject I wanted to follow deeper.

In the fourth year I continued reading algebraic geometry. I attended a PhD level course on algebraic geometry following parts of the *Algebraic Geometry* book of R. Hartshorne. This was much more abstract than the previous book and was based on Grothendieck's approach to algebraic geometry. Grothendieck became a great source of inspiration for me due to his amazingly deep mathematics and also his life story.

One problem with reading basic algebraic geometry books and attending first courses is that it is not clear where the subject is heading. One learns a lot of notions and tools but it is hard to see what that is all about. In the book of Hartshorne there is a subsection titled "What is algebraic geometry?" I read it numerous times just to get a sense of direction. The piece basically describes the classification theory of algebraic varieties starting with birational geometry and heading towards moduli theory. It is then perhaps not a surprise that I ended up working in birational geometry in which one has a clear impression of what one is trying to do, and some of my work in recent years is in the direction of constructing moduli spaces.

In the third and fourth years I skipped many lectures as I was more comfortable reading by myself. The lecturers got used to it after complaining for sometime. Still it was nice to communicate with some of the professors after lectures, for example, with A. S. Deh-Abadi, A. Chademan, Sh. Shahabi, S. Yasemi.

In the fourth year I did some research in elementary topology by investigating the space of continuous maps between topological spaces by defining some kind of topology, and wrote an article. After discussions with A. Chademan I learnt that related studies had already been done using a different kind of topology, that is, that of the compact-open topology.

Given my view of mathematics, I was not a fan of mathematics competitions. I was interested in research type problems rather than competition type problems. Our department participated in the national mathematics competition each year but I did not want to participate. In the fourth year, however, one day I was talking to a graduate student friend about the competition. I told him my reason for not participating. He told me that I should reconsider because getting a prize in such competitions can be very

helpful, for example, for getting admissions from universities abroad. Following his advice I joined the competition team. Eventually I got a gold medal in the national competition. Out team hence was selected to participate in the international version of the competition. That year the competition was in London.

Life in undergraduate was tough. We spent virtually all of our time either on the campus or in the dormitories. We lived in rooms usually with two-three other students. The main source of entertainment was students saying jokes or doing stupid things. I and most of my friends had very little money. During the four years, I recall only once visiting a coffee shop outside the campus and even then a friend paid for the tea as I could not afford it.

Figure 2. On a trip to Isfahan, 1999.

Immigration

To participate in the International Mathematics Competition for university students I travelled to London in July 2000. I got a third prize (similar to a bronze medal) as I was not well-prepared. For various reasons I stayed in London after the competition and applied for asylum. It took around one year for the UK government to decide on my case and grant me residence. I was sent to Nottingham where I lived in a house with few other refugees.

During that year I continued reading mathematics as before, mostly algebraic geometry. It was natural to get in touch with the mathematics department in Nottingham which I did with the help of a new Kurdish friend, Kambiz Fathi, who was a PhD student in the astrophysics department located in the same building as the school of mathematics. In the first meeting I talked to Ivan Fesenko and John Cremona who were eager to help both of whom are number theorists close to algebraic geometry. They suggested that I read a book by Silverman, *Rational Points on Elliptic Curves*. I did read the book and met them again who seem to be happy. Despite their eagerness to accept me as PhD student in the department, it soon became clear that I could not be given a scholarship due to my residency status. I got in touch with other universities including Cambridge, Warwick, and Bath but I could not be considered for the same reason.

Life during that year was tough. It was really stressful to wait for the asylum decision whose outcome would determine my life. A negative decision would have been disastrous. Each week we were given 10 British Pounds in cash and under 30 Pounds in vouchers to buy food and other necessities. The vouchers were accepted in certain shops such as supermarkets where some of the cashiers did not know what they were so would have to stop the line until they could speak to a supervisor.

Fortunately after about one year I was given permission to stay in the country. One of the very first things I did was to change my name to Caucher Birkar. I wanted a name that reflects who I am as a disciple of mathematics and as a Kurd. Caucher means a person who migrates from place to place and Birkar simply means mathematician. Both together can be interpreted as “an explorer in the world of mathematics” or as “a migrant mathematician”.

PhD

Soon after my residency was resolved I got an EPSRC scholarship to start PhD in Nottingham under the supervision of Ivan Fesenko who supported me in many ways during and after PhD. Ivan is a Russian mathematician educated in Saint Petersburg, Russia, so knew a great deal about algebraic geometry. He proposed a project at the interface of algebraic geometry and non-standard analysis. The idea was to apply the techniques of the latter to the former. I thought about this for a few months and wrote an article.

Knowing that I was determined to do algebraic geometry rather than number theory, Ivan suggested that I attend a one month program in Warwick University on birational geometry. This was in early 2002. Ivan also suggested that I attend the following five months program at the Newton Institute in Cambridge on birational geometry and other topics. This did not seem to be possible but due to Ivan’s insistence the organisers agreed that I attend the program with no financial support or office space. I rented a room in Cambridge and attended the program especially the birational geometry part which was focused on understanding a recent article of Vyacheslav V. Shokurov on existence of four dimensional flips. I tried hard to read Shokurov’s paper. As a result I learnt a lot about the finer techniques of birational geometry.

Figure 3. With Ivan Fesenko, Nottingham, around 2003.

At the end of the program Shokurov also came to Cambridge so I met him for the first time. I asked him if I could visit him in Johns Hopkins. I did visit him one year later in Spring 2003 and he kindly agreed to be my second PhD advisor.

In the first two years of PhD I spent sometime thinking about the termination of flips conjecture. I was not able to make much progress. The conjecture is too difficult and still open even in dimension four although some crucial cases are now known in any dimension. In the third year I visited Shokurov in Autumn 2003 and stayed in Baltimore for five months where I did a good proportion of my PhD thesis on the theory of complements on Fano varieties. The main result was a proof of the boundedness of ϵ-log canonical complements on surfaces that are relatively Fano over a base. This in particular gave a new proof of Alexeev's result on boundedness of ϵ-log canonical Fano surfaces.

Figure 4. Nottingham, around 2003.

The thesis outlined a program of Shokurov on various conjectures in birational geometry such as boundedness of complements, the Borisov-Alexeev-Borisov conjecture which concerns boundedness of ϵ-log canonical Fano varieties, ACC for minimal log discrepancies, termination of flips, etc. Learning the theory of complements proved to be valuable for works I did many years later.

Toward the end of my PhD I discovered a connection between log canonical thresholds and termination of log flips. I did not include it in my thesis as I wanted to investigate the connection further. I did publish it later which became my first published paper [7]. In the Autumn 2004 I finished my PhD. The PhD examination unfortunately did not go as smooth as it should have been due to conflict of interests not related to the mathematical contents of the thesis.

In Spring 2004, I visited Shokurov in Steklov Mathematical Institute in Moscow for two months. During these months I stayed at the main building of Moscow State University. I visited again a year later. Visiting places and attending conferences was a good way to meet people and make good friends.

During PhD I just could not focus on work in the shared office provided. Instead I used to work in libraries, common rooms, etc. With the help of Ivan and another friend, Nikolaos Diamantis, I could work in some offices on my own at least for short periods.

Figure 5. With Vyacheslav V. Shokurov, Baltimore, around 2003.

After PhD

After finishing PhD I moved to Warwick University to take up a postdoctoral research fellowship. Miles Reid and some younger algebraic geometers were there. After one year and a few months I moved to Cambridge University in April 2006 with the same research fellowship and with the help of Burt Totaro.

During the postdoc I continued working on the termination conjecture, the minimal model conjecture, and related problems. I was especially determined to dig deeper into the connection between

thresholds and termination. I realised that it was possible to actually avoid using log canonical thresholds if one only deals with the minimal model conjecture and some special cases of the termination conjecture which was enough for many purposes. Along the way I learnt from James M^{c}Kernan that he and Paolo Cascini and Christopher Hacon were working on similar problems. They had made more progress but kindly agreed to join forces and eventually publish the works in a joint paper which came to be known as the BCHM [8]. The paper contained proofs of the existence of minimal models for varieties of general and finite generation of canonical rings together with various other related results. The paper relies on earlier work of Hacon and M^{c}Kernan on existence of pl flips which in turn relies on ideas of Shokurov on existence of flips.

In the Autumn 2007 I was hired as a lecturer in Cambridge. Lecturer is more or less like a tenured position in the US. I had taught a couple of graduate courses during postdoc so it was relatively easy to get used to teaching in addition to research. There are three academic terms in Cambridge each year but they are short, each around two months. This proved quite helpful as I could freely travel in the non-term time. Research-wise I continued working on the numerous open problems in birational algebraic geometry. I talked a lot with Burt Totaro both for mathematics and for socialising until he left some years later. I also enjoyed talking with few other colleagues whenever possible, in particular, John Coates.

I spent the academic year 2010-2011 in Paris at the Jussieu Mathematics Institute, with the support of the Fondation Mathématiques de Paris, where I did some work on understanding singularities and related topics [5].

In several work on minimal models, Iitaka conjecture for fibrations, etc, I came across varieties that are "polarised" with a nef divisor, e.g. [6]. A nef divisor on a projective variety is a divisor which intersects every curve non-negatively. One can think of it as a very weak variant of an ample divisor.

On a visit to Singapore in 2014, De-Qi Zhang reminded me of a problem about effectivity of pluricanonical systems on varieties with non-negative Kodaira dimension. After working on the problem for sometime it became clear that polarised varieties which I had thought about earlier were key to this problem. By generalising the notion of polarised variety to generalised polarised variety we were able to essentially reduce the problem to the case of Kodaira dimension zero [9].

I felt that the new theory of generalised polarised varieties could also be applied to problems about Fano varieties, in particular, the boundedness of complements conjecture. Indeed, it turned out to be an important ingredient to tackle the conjecture. However, it became clear that more was needed to solve the conjecture so I turned to other techniques for proving effective birationality of anti-pluricanonical systems on ϵ-log canonical Fano varieties by further developing ideas of Hacon, M^{c}Kernan, and Xu in their work on effective birationality for varieties of general type. Overcoming numerous difficulties I completed the proof of the boundedness of complements conjecture around Summer 2015.

While proving boundedness of complements, I was convinced that I could also approach the so called Borisov-Alexeev-Borisov (BAB) conjecture. I worked out all the steps except one step, about singularities of anti-canonical divisors, which I left to do last and I thought it was simple. But in the end I realised that this last step is not simple at all and seemed to require quite new techniques. As a result I did not publish the proof of boundedness of complements in 2015. But many months later, in 2016, eventually I posted the proof on arXiv [4]. Around six months later I completed the proof of BAB conjecture by working out the mentioned last step which indeed required quite different techniques, and published it on arXiv [3].

This is not the first time that Fano varieties have been at the centre of a breakthrough in algebraic geometry. They played a key role in extending birational geometry of the Italian algebraic geometry school from dimension two to higher dimension starting with Fano and others in the first half of the 20th century. They were also at the centre of the explosion of activities in the 1970's, in particular, in the work of Iskovskikh and Shokurov, and of course Mori.

For the works mentioned above, and others not mentioned, I was lucky enough to be awarded a Fields Medal in the 2018 International Congress of Mathematicians in Brazil. It also recognises work done by my colleagues in my field over the last few decades.

As everyone knows the medal was stolen shortly after awarded but the International Mathematical Union kindly handed me a replacement a couple of days afterwards. The incidence was reported widely around the globe which did not surprise me. It is a pity that so much attention is given to a relatively small incidence yet much more fundamental issues are barely mentioned in the media.

The above results on Fano varieties and their proofs are related to many other problems in algebraic geometry, e.g. singularities, birational automorphism groups of varieties, K-stability of Fano varieties, moduli, etc. In the last few years I have come to the realisation that they form part of a much larger and more ambitious program on log Calabi-Yau fibrations, see [2] for some aspects. A log Calabi-Yau fibration is a pair which is relatively Calabi-Yau over some

Figure 6. ICM 2018. Photo by Pablo Costa.

base. These are structures which can be viewed from so many different points of view ranging from geometry, arithmetic to physics, etc. For example, one of my latest work establishes boundedness of polarised semi-log canonical Calabi-Yau pairs which, combined with the moduli theory of stable pairs, implies existence of projective coarse moduli spaces for polarised Calabi-Yau and Fano pairs [1].

I have been working in a field which has attracted brilliant mathematicians whose work I have relied on. This includes Cascini, Hacon, Kawamata, Kollár, M^{c}Kernan, Miyaoka, Mori, Reid, Shokurov, Xu, and Fano, Hironaka, Iitaka, Iskovskikh from older days, and many more that is not possible to list here.

Life as a Mathematician

Working as a mathematician in research active universities involves various tasks. Research is a big part of the work which consumes much of the time and energy of a mathematician. Teaching is another part of the job. Depending on what one teaches it can also take a lot of time. I made it a point to teach different kind of courses especially graduate courses which can consume a lot of time. For example, I have taught graduate courses on algebraic geometry, birational geometry, the Weil conjectures, Hilbert schemes and related topics, arithmetic geometry, moduli theory, etc. In Cambridge being involved in undergraduate examinations also takes time.

Service is another part of the job. This usually comes in the form of serving on a committee. Other forms of service include organising seminars and conferences, refereeing articles and grant proposals, editing journals, etc.

To stay up to date one needs to travel and talk to other mathematicians. This also takes a lot of time and energy. I have travelled my fair share, especially to East Asia. I visited Taipei regularly for some years to meet Jungkai A. Chen. In recent years I have been visiting Beijing to meet Shing-Tung Yau.

The great thing about an academic job is the freedom to think about fundamental scientific problems. It is great when one feels the beauty and pleasure of discovery, of unlocking the mysteries of nature and life. It is much more than just a job. Sometimes it is hard to get work out of our head even if we actually try to stop working.

As fan as it is to be an academic the truth is that it can be a stressful job. There are just too much demand on our time. We do lots of work that is not even paid for, for example, we do not get paid for refereeing and editing journals (at least this is true in mathematics). Although we take pride in doing such work as service to the community but the point is that they take time. It is not uncommon for people to spend time in the evening or weekend to do part of their job in which case they have less time to take care of themselves and their families. In my case I have spent hundreds, more likely thousands, of hours of my "free time" on work related tasks.

As for working habits, the best for me is to do research three or so hours in the morning, then have a break and then do research another three or so hours (this is only research and does not include other work). However, due to various commitments it is not always possible to afford this luxury.

After receiving the Fields Medal, my life has changed in a few ways. First, I was happy that my work and the advances in my field were recognised in the wider mathematical community. Second, I have the opportunity to meet many interesting people with different background whom I would have otherwise not met. Third, it is a bit easier now for me to work on popularising mathematics and science, something that I have been doing for some years, especially improve education among the Kurds, for example, by giving public talks or using social media – one of the latest projects that I am working on is making a Kurdish video series on my own titled "The world of mathematics" examining many aspects of the mathematical sciences and connections with life and nature in general.

Family

I met my wife, Tarn, while doing PhD in Nottingham. She was studying economics. We got married and had our son, Zanko, some years later. I have made it a point to spend as much time as possible with my family and take a leading role in the development and education of Zanko. He is passionate about nature which resonates well with my own interests. His interests change time to time ranging from astronomy

to animal life to human anatomy. We play various fun games often including some scientific facts or experiments.

Another daily activity is storytelling. I tell Zanko stories, mostly home-made, which are meant to be fun but also educational. For example, in the story there may be a scenario in which one comes across a cave or a building which requires answering a question to open the door, e.g. how much is the sum of eight and seven. I also often let Zanko take part in inventing parts of the story. We are usually the main characters in most of these stories.

Other Interests

Other than mathematics I have followed other passions in my life, mainly for curiosity or fun. Here I discuss a few but not all.

Psychology I like psychology, an interest I have maintained since high school. I have always been very curious about my own and other people's thought processes. Why do people think in a certain way? I came to be convinced long ago that people's decisions are not simply based on rational thinking as they may believe but often strongly influenced by other factors that they may take for granted, e.g. social, cultural, or biological. For example, the media has a strong influence on how people think about a particular issue – even whether the issue is worth considering.

Psychology is important not only on a personal level but also on the society level. For example, I think lack of confidence, which is a psychological state, is one of the top reasons preventing progress in many economically and technologically underdeveloped societies around the world.

Music Another interest is music. I listened to music a lot in high school as I did later on. I did not have any music instrument so did not learn to play music until much later. I was fascinated by the infinite possibilities of different sounds and different ways in which they can be combined to create music. This is somehow similar to the endless possibilities of mathematical ideas and structures. These days I play drum and flute on my own and sometimes try to compose music on an electronic keyboard or just a computer.

I usually listen to Kurdish music, world music, and other genres. I especially like when the lyric is in a language that I do not understand! The reason is that I think very often lyrics simply ruin a song as they do not go well with the music. Not understanding the lyric one can focus on the beauty of the actual music and the vocals.

Biology Evolution and the inner workings of living organisms are fascinating. The diversity of life, countless ways of survival and adaptations are amazing. Take for example how the human hearing works. Sound made through vibrations of the air is collected by the outside of the ears and channelled through the ear air-canal reaching the eardrum which vibrates. The eardrum in turn vibrates three little bones which amplify the sound and in turn vibrate the liquid inside the cochlea in the inner ear containing bundles of hair whose vibration produces electrical signals in the nerves connecting the ear with the brain which then interprets the signals as sound. The little bones mentioned originally evolved for a completely different purpose and reused in the hearing system. Understanding biological systems involves not only biology but also chemistry, physics, mathematics, etc.

Literature Kurdish classical literature consists mostly of some forms of poetry. Some of these are world class, e.g. poems of 19th century poet Nali, but not known to the world for various reasons. There still are many people these days who are interested in poetry one way or another.

In high school I also got interested in poetry, in writing it rather than just reading classical work. I wrote a number of poems in a notebook which unfortunately was lost like almost everything else from that period. These days I am more interested in writings that use colourful language by combining words and expressions like composing music.

Society and Globalisation In my lifetime I have witnessed rapid changes of various aspects of the Kurdish society. Similar changes occur in many societies around the world so this is not a particularly Kurdish issue but rather a global one. One can see changes in many aspects of life from people's values and self-image to social connections, dressing, housing, food, etc. Some changes are positive and some negative or simply unnecessary. Modernisation and globalisation improve certain aspects, e.g. access to education and medicine, but also often destroy other aspects.

Let's focus on some of the changes in my home town compared to say 40 years ago. Education and medicine are more accessible, technology is way more present, disable people are treated better, etc. These are all positive. On the other hand, Kurdish language has suffered a lot as people are slowly replacing many words and expressions with those coming from other languages; Kurdish architecture which has been around for thousands of years is fast disappearing; confidence is way lower than before which is ironically partially due to education because the more people learn about advanced and powerful societies the less they think of themselves. These are all negative changes.

The point I want to make is that it is possible to progress without destruction. Take Kurdish architecture for example. Instead of replacing it entirely with other styles we can modify it, take it to another level while keeping some of the traditional characteristics. This way we keep an art form alive which in the long run benefits the whole world.

I believe cultural diversity is important for human survival just as biodiversity is important for survival of species and ecosystems. Humanity will not benefit from cultures disappearing although it may benefit dominant cultures in the short run.

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