TOPS SELECTION – ANNOUNCEMENT

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OPEN PROBLEMS, OLD AND NEW

In 1900, David Hilbert introduced the Hilbert Problems, a set of 23 significant unsolved challenges in mathematics. These problems notably influenced the evolution of 20th-century mathematics, and a number of them even spurred the creation of new mathematical fields. The broad scope of these problems displayed the unity and connections across various areas of mathematics. While many have been solved, the remaining ones continue to motivate today's mathematicians. In a similar vein, the 120 problems in geometry put forward by Shing-Tung Yau in 1982 have greatly influenced the subsequent development of the field of geometry.
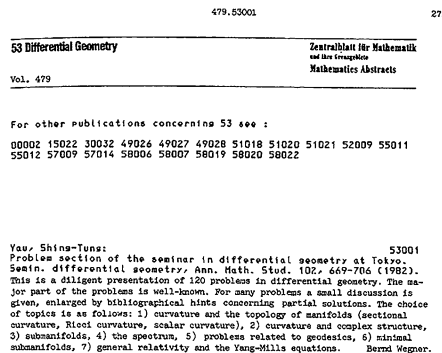

Figure 1. Problem section in Seminar on Differential Geometry in [1].

In May 2023, Yau once again gave a presentation with the title "Open Problems in Geometry" at Tsinghua University, covering a list of questions and conjectures in geometry, including known evidence. Topics ranged from the Milnor Conjecture on the relationship between curvatures and fundamental groups, to the potential links between complex and symplectic structures, then to the problems related to Calabi-Yau and their relevance to general relativity, and also some open questions in deformed Hermitian-Yang-Mills equations. This lecture was hosted by the TOPS Committee of Qiuzhen College of Tsinghua University together with the Yau Mathematical Sciences Center.

Figure 2. Open Problems in Geometry by Shing-Tung Yau at TOPS seminar.

What Is TOPS? – Purpose and Goal

TOPS stands for Tsinghua Open Problems Set, and its organizers are the Yau Mathematical Sciences Center of Tsinghua University and the Qiuzhen College of Tsinghua University. The genesis of TOPS stems from Professor Yau's understanding of the importance of "asking questions" in the discipline of mathematics. In a public lecture themed "The Pursuit of Knowledge and Inquiry" (学"问") at the Humanities Tsinghua Forum in 2022, Yau emphasized the critical role that asking questions has played in the historical development of the field of mathematics and other sciences. The importance of asking questions is also embedded in the educational philosophy of Qiuzhen College at Tsinghua University: studying the history of mathematics, bridging past and present, expanding horizons; solidifying foundations, reviewing important mathematical work from all eras, understanding the most critical ideas and tools; asking profound questions, developing new tools, advancing the discipline of mathematics.

In this context, asking profound questions is seen as an integral part of learning and research. It's viewed as an essential characteristic of an outstanding mathematician. Recognizing the importance of this practice, Tsinghua University launched the Tsinghua Open Problems Set and the Tsinghua Open Problems Seminars in the spring of 2023, aimed at cultivating students' ability to pose insightful questions.

In the initial phase, the Tsinghua Open Problems Set plans to invite leading researchers from 12 crucial fields of mathematical sciences, both locally and internationally, to introduce their subjects to students at Qiuzhen College. The setup was conceived as lunch seminars, with each presenter giving at least a 45-minute presentation. Their lectures would focus on significant ideas and open problems within their respective fields.

Figure 3. Audience.

The importance of collecting and summarizing frontier problems from various important research areas and providing students with ample research materials is undeniable. Concurrently, TOPS also offers a space for researchers to test their ideas, refine their arguments, and receive constructive feedback. Moreover, the Tsinghua Open Problems Set serves as a platform for interaction between scholars and students. Students interested in a specific direction or problem are encouraged to follow up and discuss with the speakers. The content of each presentation, along with the associated questions, will eventually be summarized into survey papers. These papers will be published in the TOPS Selection of the ICCM Notices.

In parallel, TOPS encourages students to ask questions and participate actively. The TOPS Selection also plans to regularly publish noteworthy questions posed by students, fostering a culture of inquiry and intellectual curiosity.

Figure 4. Mao Sheng.

Past Themes in TOPS

The TOPS seminars commenced in March 2023, during the visits of Professor Kenji Fukaya and Professor Maxim Kontsevich to Tsinghua University and Beijing Institute of Mathematical Sciences and Applications. In their inaugural seminar, Fukaya and Kontsevich sparked an intellectual journey that began with open problems readily comprehensible to students, but ultimately led to some profound and challenging questions in the realm of symplectic geometry, questions that even Fukaya considers to be formidable within his field. The second session of the TOPS seminar unfolded with the insightful presentations of Fukaya and Kontsevich, joined by Mao Sheng and Hossein Movasati. They delved into topics related to Hodge Conjecture and the Tate Conjecture, providing unique viewpoints on these complex and challenging areas in mathematics.

April saw the TOPS series turn its focus to number theory. Featured leading researchers Chung Pang Mok, Ivan Fesenko, Xu Shen, and Yichao Tian enlightened the audience on an array of subjects, spanning recent developments in algebraic number theory, Langlands programs, and arithmetic geometry. Each presentation was carefully curated to ensure students could not only grasp the significance of the topics but also appreciate their inherent beauty. Additionally, each speaker proposed several problems related to their respective topics, which could serve as valuable starting points for interested students.

Figure 5. Kenji Fukaya.

During the month of May, the TOPS series shifted its focus to geometry and leading figures such as Gao Chen, Jixiang Fu, Shing-Tung Yau, and Lars Andersson. The topics covered were diverse, ranging from the Hodge Conjecture to geometric analysis and intersections with mathematical physics. Additionally, Yau shared his collection of geometry problems.

Figure 6. Maxim Kontsevitch.

The TOPS Selection

The ICCM Notices TOPS Selection will publish survey articles summarizing relevant questions from the TOPS seminars, as curated by students from Qiuzhen College, as well as excellent questions posed and answered by the students. Concurrently, the field of mathematics features many articles and books dedicated to open problems. The ICCM Notices TOPS Selection will not only feature TOPS seminar content but also publish informative articles about open problems. In the inaugural issue, the TOPS Selection will feature "Some open problems in birational geometry" by Caucher Birkar, a comprehensive overview of challenges and pertinent literature intended for a specialized audience engaged in this field.
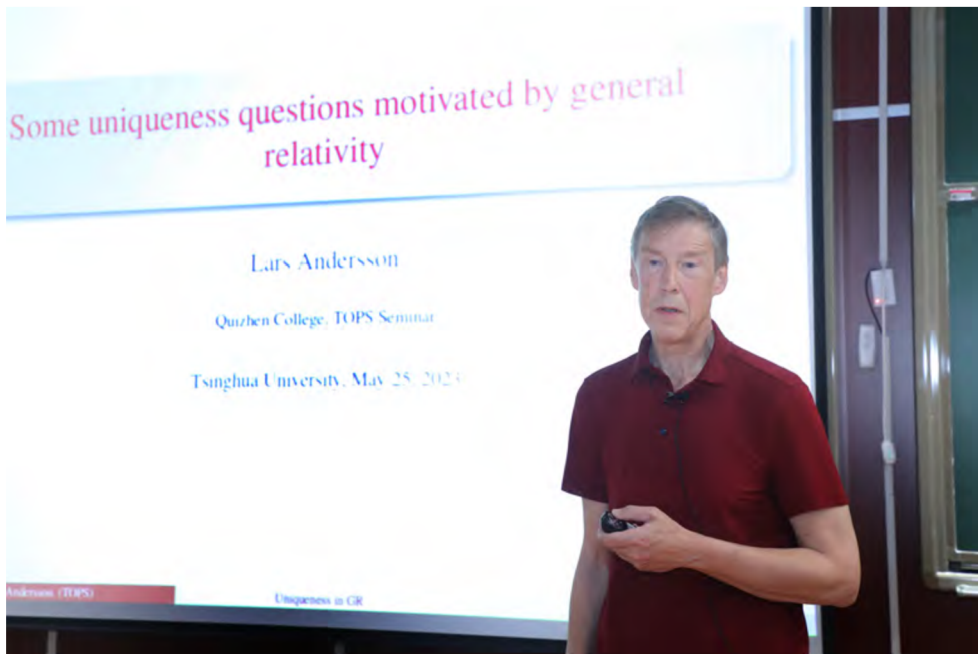

Figure 7. Lars Andersson.

References

[1] Shing-Tung Yau. Problem section of the seminar in differential geometry at Tokyo. Semin. differential geometry, Ann. Math. Stud. 102, 669–706 (1982), 1982. MR0645762

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