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Removing parametrized rays symplectically

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Let (M, ω) be a symplectic manifold. Let $[0, \infty) \times Q \subset \mathbb{R} \times Q$ be considered as parametrized rays $[0, \infty)$ and let $\varphi : [-1, \infty) \times Q \to M$ be an injective, proper, continuous map immersive on $(-1, \infty) \times Q$. If for the standard vector field $\frac{\partial}{\partial t}$ on $\mathbb{R}$ and any further vector field ν tangent to $(-1, \infty) \times Q$ the equation $\varphi^*\omega(\frac{\partial}{\partial t}, \nu) = 0$ holds then M and $M \setminus \varphi([0, \infty) \times Q)$ are symplectomorphic.

The question which subsets N of a symplectic manifold M can be chosen such that M and $M \backslash N$ are symplectomorphic has been treated in particular for $M = \mathbb{R}^{2n}$ a time ago already, see e.g. [Gro85, McD87, MT93, Tra93]. More recently, X. Tang showed that for a general manifold M the subset N can be chosen to be a ray if the ray possesses a "wide neighborhood" ([Tan20]). Roughly speaking a ray is a 2-ended connected non-compact 1-dimensional local submanifold whose one end closes up inside M while at the other end the embedding is proper. In this paper an extension to higher dimensional sets regarded as parametrized rays is provided. While for those higher dimensional sets a condition is needed, this condition is trivially fulfilled for an isolated ray as treated in [Tan20].

In order to state the theorem precisely let $\frac{\partial}{\partial t}$ denote the standard vector field on $\mathbb{R}$, i.e. whose flow consists of translations.

Theorem. *Let (M, ω) be a symplectic manifold, Q some manifold and the map $\varphi : [-1, \infty) \times Q \to M$ be injective, proper and continuous such that $\varphi|_{(-1,\infty)\times Q}$ is immersive. If the equation*

$$\imath_{\frac{\partial}{\partial t}} \varphi^* \omega = 0 \tag{1}$$

holds on $(-1, \infty) \times Q$ then M and $M_0 = M \backslash \varphi([0, \infty) \times Q)$ are symplectomorphic.

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Additionally this symplectomorphism can be chosen to be the identity outside some selected neighborhood of $\varphi([0,\infty) \times Q)$.

During the final preparations of this paper the author learned about the recent e-print of Y. Karshon and X. Tang ([KarTan21]) where similar statements are proven using time-independent flows.

For simplicity, the set $[-1,\infty) \times Q$ will be seen as a subset of M as the map φ is assumed to be injective, proper and continuous.

Each ray $[0,\infty) \times \{q\}$ has an extension $[-1,0) \times \{q\}$. The proof will construct a diffeomorphism $\theta : [-1,0) \times Q \to [-1,\infty) \times Q$ mapping for each $q \in Q$ the extension $[-1,0) \times \{q\}$ diffeomorphically to the corresponding extended ray $[-1,\infty) \times \{q\}$. This map θ extends to the desired diffeomorphism $\psi : M_0 \to M$ which is injective, surjective and satisfies $\psi^*\omega = \omega$, hence is a symplectomorphism.

The idea of the proof is as follows. A smooth curve $s \mapsto \theta_s$ for $s \in \mathbb{R}$ of diffeomorphisms $\theta_s : [-1,\infty) \times Q \to [-1,\infty) \times Q$ is constructed which will converge on $[-1,0) \times Q$ to a diffeomorphism $\theta : [-1,0) \times Q \to [-1,\infty) \times Q$ for $s \to \infty$. This curve θ_s of diffeomorphisms in turn will be extended to a curve $s \mapsto \psi_s$ of symplectomorphisms $\psi_s : M \to M$. This latter curve will converge on M_0 to the desired symplectomorphism $\psi : M_0 \to M$ for $s \to \infty$. The diffeotopy θ_s will be constructed as the flow of a time-dependent vector field ζ_s extending to a time-dependent Hamiltonian vector field ξ_s whose flow is ψ_s.

This time-dependent vector field ξ_s will be constructed to satisfy

(ξ1) the set

$$\bigcup_{s' \in [s-1,s+1]} \left(\operatorname{int}\left(\{m \in M \mid \xi_{s'}|_m = 0\}\right) \right)^c$$

is relatively compact for all $s \in \mathbb{R}$,
where by "int" the topological interior is denoted and the complement by "superscript c".

(ξ2) for all compact subsets $K \subset M_0$ there is $s_K \in \mathbb{R}$ such that $\psi_s|_K = \psi_{s'}|_K$ for all $s, s' \geq s_K$ and

(ξ3) for each compact set $L \subset M$ there exists $\sigma_L \in \mathbb{R}$ such that

$$\psi_\sigma|_{\psi_{\sigma_L}^{-1}(L)} = \psi_{\sigma'}|_{\psi_{\sigma_L}^{-1}(L)}$$

for all $\sigma, \sigma' \geq \sigma_L$.

Property ($\xi 1$) provides integrabilty of ξ_s, i.e. the existence of diffeomorphisms $\psi_s : M \to M$ as the flow of ξ_s. Furthermore Property ($\xi 2$) will ensure that the curve of diffeomorphisms ψ_s becomes locally stable on M_0 for $s \to \infty$ and therefore converges to a limit $\psi : M_0 \to M$ which is an injective immersion automatically. Since ξ_s is a Hamiltonian vector field such that $\psi_s^* \omega = \omega$, the limit ψ is still satisfying $\psi^* \omega = \omega$ while finally Property ($\xi 3$) will provide surjectivity of this limit ψ.

Proof. In the first part of the proof a suitable diffeotopy

$$\theta_s : [-1, \infty) \times Q \to [-1, \infty) \times Q \qquad \text{with } s \in \mathbb{R}$$

is constructed which in turn is constructed from a diffeotopy τ_s of $[-1, \infty)$.

For later extension to the ambient space the maps θ_s and θ will equal the identity around $\{-1\} \times Q$ and therefore τ_s and τ shall equal the identiy around $\{-1\}$. So fix $b \in (-1, 0)$ and a diffeomorphism $\tau : [-1, 0) \to [-1, \infty)$ with $\tau|_{[-1,b]} = \mathrm{id}_{[-1,b]}$. There is a diffeotopy τ_s, i.e. a smooth curve $s \mapsto \tau_s$ of diffeomorphisms $\tau_s : [-1, \infty) \to [-1, \infty)$, such that

($\tau 1$) $\tau_s = \mathrm{id}$ for all $s \leq 0$,

($\tau 2$) $\tau_s|_{[-1,b]} = \mathrm{id}_{[-1,b]}$ for all $s \in \mathbb{R}$,

($\tau 3$) for all $s \in \mathbb{R}$ the set

$$\bigcup_{s' \in [s-1, s+1]} \left(\, \mathrm{int} \left(\{ t \in (-1, \infty) \mid \tau_{s'}(t) = t \} \right) \, \right)^c$$

is relatively compact in $(-1, \infty)$ and

($\tau 4$) for each compact subset $A \subset (-1, 0)$ there is $s_A \in \mathbb{R}$ such that $\tau_s|_A = \tau|_A$ for all $s \geq s_A$.

Property ($\tau 3$) will provide Property ($\xi 1$) of the extended Hamiltonian vector field ξ_s while Property ($\tau 4$) will be needed to ensure the extension ξ_s to satisfy both Properties ($\xi 2$) and ($\xi 3$) later, needed for the flow becoming locally stable and the limit being surjective.

Property ($\tau 1$) is only used to define θ_s later via a time shift.

Now the diffeomorphism $\theta : [-1, 0) \times Q \to [-1, \infty) \times Q$ can be defined from $\tau : [-1, 0) \to [-1, \infty)$ in the most naive way by $\theta(t, q) = (\tau(t), q)$. However, if Q is not compact, defining the diffeotopy θ_s° in the same naive way would result in that θ_s° differs from the identity on a non-compact set contradicting Property ($\xi 1$). Instead, θ_s° is defined using a time shift. Choose a function $\rho : Q \to (0, \infty)$ such that the set $\{q \in Q \mid \rho(q) \leq c\}$ is compact for each $c \in \mathbb{R}$. Define θ_s° by

$$\theta_s^\circ(t, q) = (\tau_{s-\rho(q)}(t), q) \ .$$

Denoting $\pi_Q : \mathbb{R} \times Q \to Q$ the projection the maps θ_s° satisfy $\pi_Q \circ \theta_s^\circ = \pi_Q$ while for fixed q and s the map $t \mapsto \tau_{s-\rho(q)}(t)$ is a diffeomorphism of $[-1, \infty)$. Thus θ_s° is a diffeomorphism. By construction it satisfies

($\theta 1$) $\theta_s^\circ = \mathrm{id}$ for all $s \leq 0$,

($\theta 2$) $\theta_s^\circ|_{[-1,b] \times Q} = \mathrm{id}_{[-1,b] \times Q}$ for all $s \in \mathbb{R}$,

($\theta 3$) for all $s \in \mathbb{R}$ the set

$$\bigcup_{s' \in [s-1, s+1]} \left(\ \mathrm{int} \left(\{(t, q) \in (-1, \infty) \times Q \mid \theta_{s'}^\circ(t, q) = (t, q)\} \right) \ \right)^c$$

is relatively compact in $(-1, \infty) \times Q$ and

($\theta 4$) for each compact set $A \subset [-1, 0) \times Q$ there is $s_A \in \mathbb{R}$ such that $\theta_s^\circ|_A = \theta|_A$ for all $s \geq s_A$.

Later, the Hamiltonian vector fields which define the Hamiltonian flow will be modified using cut off functions (see ($\chi 2$) and ($\chi 3$) below). For this, the flow is required to be static in a neighborhood of time $s \in \mathbb{Z}$. This is done by deforming the time of the diffeotopy θ_s°. For this, choose an increasing smooth map $\kappa : \mathbb{R} \to \mathbb{R}$ and $\delta > 0$ such that $\kappa|_{[n-\delta, n+\delta]} = n$ for all $n \in \mathbb{Z}$ and define the diffeotopy $\theta_s = \theta_{\kappa(s)}^\circ$ satisfying likewise all the above Properties ($\theta 1$)-($\theta 4$). Denote ζ_s the time-dependent vector field whose flow is θ_s. The property $\pi_Q \circ \theta_s = \pi_Q$ shows that the time-dependent vector field ζ_s points in the direction of the rays, i.e. there is a time-dependent function $\lambda_s : [-1, \infty) \times Q \to \mathbb{R}$ such that

$$\zeta_s = \lambda_s \cdot \frac{\partial}{\partial t} \ . \tag{2}$$

By construction ζ_s satisfies the following properties.

($\zeta 1$) $\zeta_s = 0$ for all $s \leq 0$ and for all $s \in [n - \delta, n + \delta]$ for each $n \in \mathbb{Z}$,

($\zeta 2$) $\zeta_s|_{[-1,b] \times Q} = 0$ for all $s \in \mathbb{R}$,

($\zeta 3$) for each $s \in \mathbb{R}$ the set

$$\bigcup_{s' \in [s-1,s+1]} \Big(\operatorname{int}\big(\{(t,q) \in (-1,\infty) \times Q \mid \zeta_{s'}|_{(t,q)} = 0\}\big) \Big)^c$$

is relatively compact in $(-1,\infty) \times Q$ and

($\zeta 4$) for each compact subset $A \subset [-1,0) \times Q$ there is an $s_A \in \mathbb{R}$ defining $B = \theta_{s_A}(A)$ such that $\zeta_s|_B = 0$ for all $s \geq s_A$.

Denote $C_s = \{(t,q) \in [-1,\infty) \times Q \mid \zeta_{s'}|_{(t,q)} = 0 \text{ for all } s' \geq s\}$ the set of those points in $[-1,\infty) \times Q$ the flow has finally stopped at at time at most s. Then the above properties imply

$$\begin{gathered} \bigcup_{s \in \mathbb{R}} \operatorname{int}(C_s) = [-1,\infty) \times Q \qquad \text{and} \\ \bigcup_{s \in \mathbb{R}} \theta_s^{-1}(\operatorname{int}(C_s)) \supset [-1,0) \times Q \end{gathered} \tag{3}$$

This can be seen as follows. Let B be an open subset of $[-1,\infty) \times Q$ such that $\bar{B}$ is compact and $\operatorname{int}\big(\bar{B}\big) = B$. Property ($\theta 4$) states that for $A = \theta^{-1}(\bar{B})$ there is $s_A \in \mathbb{R}$ such that $\theta_s|_A = \theta|_A$ for all $s \geq s_A$ hence $\zeta_s|_B = 0$ for all $s \geq s_A$, i.e. $B \subset \operatorname{int}(C_{s_A})$. Exhausting $[-1,\infty) \times Q$ by such sets B yields the first statement of (3). The second equality of (3) follows from

$$\theta^{-1}(\operatorname{int}(C_s)) \subset \theta_s^{-1}(\operatorname{int}(C_s))$$

and using the first statement of (3)

$$\begin{aligned} [-1,0) \times Q &= \theta^{-1}\Big(\bigcup_{s \in \mathbb{R}} \operatorname{int}(C_s)\Big) \\ &= \bigcup_{s \in \mathbb{R}} \theta^{-1}(\operatorname{int}(C_s)) \quad \subset \quad \bigcup_{s \in \mathbb{R}} \theta_s^{-1}(\operatorname{int}(C_s)) \ . \end{aligned}$$

The goal of this second part is to extend the time-dependent vector field ζ_s on $[-1,\infty) \times Q$ to a time-dependent Hamiltonian ξ_s on M satisfying suitable conditions.

In a first step, a time-dependent function $g_s : M \to \mathbb{R}$ with $g_s|_{[-1,\infty) \times Q} = 0$ is constructed satisfying

$$\imath_{\zeta_s} \omega|_{(-1,\infty) \times Q} = \mathrm{d}g_s|_{(-1,\infty) \times Q} \tag{4}$$

such that $\imath_{\xi_s}\omega = \mathrm{d}g_s$ defines an extension ξ_s of ζ_s. Using Equation (2), Condition (1) requested to hold in the Theorem reads

$$\omega(\zeta_s, \nu) = 0 \qquad \text{for all} \quad \nu \in T((-1,\infty) \times Q) \subset TM$$

One down to the earth way to see that g_s exists might be to see that g_s can be given in local coordinates explicitly. The global result is then easily obtained by a partition of unity.

This curve of functions g_s may not have sufficiently small support, it will therefore be cut off in the following. In detail, by construction, $\mathrm{d}g_s$ vanishes on $(-1,\infty) \times Q$ if and only if ζ_s vanishes (see (4)). Combining this with (ζ3), there is a smooth curve of smooth functions $\chi_s : M \to [0,1]$ satisfying

$$\{(t,q) \in (-1,\infty) \times Q \mid \mathrm{d}g_s|_{(t,q)} \neq 0\} \subset \{m \in M \mid \chi_s(m) = 1\} \tag{5}$$

for all $s \in \mathbb{R}$ as well as that

$$\bigcup_{s' \in [s-1,s+1]} (\ \mathrm{int}\,(\{m \in M \mid \chi_{s'}(m) = 0\})\ )^c$$

is relatively compact for all $s \in \mathbb{R}$. Now the curve $\tilde{g}_s = \chi_s \cdot g_s$ satisfies $\mathrm{d}\tilde{g}_s = \mathrm{d}g_s$ on $(-1,\infty) \times Q$, since on the one hand $\mathrm{d}\tilde{g}_s = \chi_s \mathrm{d}g_s$ on this set as g_s vanishes there and on the other hand $\chi_s = 1$ whenever $\mathrm{d}g_s$ does not vanish (see (5)). Therefore $\tilde{g}_s$ has the desired support, namely for all $s \in \mathbb{R}$

$$\bigcup_{s' \in [s-1,s+1]} (\ \mathrm{int}\,(\{m \in M \mid \tilde{g}_{s'}(m) = 0\})\ )^c$$

is relatively compact.

Starting from $\tilde{g}_s$ a time-dependent function f_s will be defined as the limit of a sequence of time-dependent functions $f_{n,s}$. Each such time-dependent function $f_{n,s}$ and f_s define a time-dependent Hamitonian vector field by

$$\imath_{\xi_{n,s}}\omega = \mathrm{d}f_{n,s} \quad \text{and}$$
$$\imath_{\xi_s}\omega = \mathrm{d}f_s$$

with flows $\psi_{n,s}$ and ψ_s respectively.

Initialize $f_{1,s} = \tilde{g}_s$ and set $f_{n+1,s} = \chi_{n,s} \cdot f_{n,s}$ for a sequence of smooth time-dependent functions $\chi_{n,s} : M \to [0,1]$ whose properties will be specified below. Since for all time-dependent functions $h_s \in \{f_{n,s}, f_s\}$ the set

$$\bigcup_{s' \in [s-1,s+1]} \left(\operatorname{int}\left(\{m \in M \mid h_{s'}|_m = 0\}\right) \right)^c \subset \bigcup_{s' \in [s-1,s+1]} \left(\operatorname{int}\left(\{m \in M \mid \tilde{g}_{s'}|_m = 0\}\right) \right)^c$$

is relatively compact for all $s \in \mathbb{R}$, the flows $\psi_{n,s}$ and ψ_s are defined for all $s \in \mathbb{R}$ globally.

In order to define $\chi_{n,s}$ let L_n be an exhausting sequence of compact subsets of M, i.e. $L_0 = \emptyset$, $L_n \subset \operatorname{int}(L_{n+1})$ and $\bigcup_{n\in\mathbb{N}} L_n = M$. In view of (3), this choice can be made such that $L_n \cap [-1,\infty) \times Q \subset \operatorname{int}(C_n)$ for all $n \in \mathbb{N}$. Analogously using (3) again, an exhaustion K_n of $M \backslash ([0,\infty) \times Q)$ is chosen such that $K_n \cap [-1,0) \times Q \subset \theta_n^{-1}(\operatorname{int}(C_n))$.

The time-dependent functions $\chi_{n,s}$ shall satisfy

(χ1) $\chi_{n,s}(m) = 1$ if $n \geq s$,

(χ2) $\chi_{n,s}(m) = 1$ if $m \in [-1,\infty) \times Q$ and $\mathrm{d}f_{n,s}(m) \neq 0$ and

(χ3) $\chi_{n,s}(m) = 0$ if $s \in [n+\delta, \infty)$ and $m \in L_n \cup \psi_{n,n}(K_n)$.

To see that this is possible note that the sets L_n and K_n have been chosen such that for all $s \geq n$

$$W_n = (L_n \cup \psi_{n,n}(K_n)) \cap [-1,\infty) \times Q \subset \operatorname{int}(C_n) \quad \text{, i.e.}$$

$$\zeta_s|_{W_n} = 0 \quad \text{for all } s \geq n \ .$$

where for $s \geq n$ the set $W_n = (L_n \cap [-1,\infty) \times Q) \cup \theta_n(K_n \cap [-1,\infty) \times Q)$ and hence depends only on θ_n and not on the extension $\psi_{n,n}$. So $\chi_{n,s}(m)$ can be chosen to vanish for $s \geq n+\delta$ satisfying (χ3) and likewise (χ1) and (χ2). For $s \in (n, n+\delta)$ the vector field ξ_s vanishes by construction such that $\chi_{n,s}$ can be chosen freely to fit the conditions for $s \leq n$ and $s \geq n+\delta$. Furthermore the sequence $\chi_{n,s}$ can be defined inductively, since the choice of $\chi_{n,s}$ does not change $\psi_{s'}$ for all $s' \leq n$.

Property (χ1) implies that

$$\begin{matrix} f_{n,s} = f_{m,s} \quad \text{and} \quad \xi_{n,s} = \xi_{m,s} \\ \text{and hence} \quad \psi_{n,s} = \psi_{m,s} \end{matrix} \qquad \text{for all} \quad n,m \geq s \ .$$

So, $f_{n,s}$ converges to f_s for $n \to \infty$, and consequently $\xi_{n,s}$ converges to ξ_s and so the sequence of corresponding flows $\psi_{n,s}$ converges to the flow ψ_s of

ξ_s. The limits satisfy

$$f_{n,s} = f_s \quad \text{and} \quad \xi_{n,s} = \xi_s \quad \text{and} \quad \psi_{n,s} = \psi_s \qquad \text{for all} \quad n \geq s \ .$$

By making use of Property ($\zeta 1$) incidentally, this condition ($\chi 3$) implies

$$\psi_s|_{K_n} = \psi_{s'}|_{K_n} \quad \text{for all } s, s' \geq n \ .$$

Thus, since K_n has been chosen to exhaust $M_0 = M \setminus ([0,\infty) \times Q)$, the diffeomorphisms ψ_s converge for $s \to \infty$ on M_0 to a diffeomorphism $\psi : M_0 \to \psi(M) \subset M$. Recall that each diffeomorphism $\psi_{n,s}$ as well as each diffeomorphism ψ_s is in fact a symplectomorphism, so the limit ψ satisfies likewise $\psi^*\omega = \omega$.

Finally the map ψ will be shown to be surjective to M. By construction $\psi|_{[-1,0)\times Q} = \theta$, so the image of θ, namely $[-1,\infty) \times Q$, is contained in the image of ψ. For a compact set L disjoint to $[-1,\infty) \times Q$ there is $n \in \mathbb{N}$ such that $L \subset L_n$. Furthermore ψ_n is a bijection of M and the restriction $\psi_n|_{M\setminus([-1,\infty)\times Q)}$ a bijection of $M \setminus ([-1,\infty) \times Q)$, i.e.

$$L \subset \psi_n(M \setminus ([-1,\infty) \times Q)) \ .$$

For each $l \in L$ there is an $m \in \psi_n^{-1}(L)$ such that

$$l = \psi_s(m) = \psi_{s'}(m) \quad \text{for all } s, s' \geq n$$

with $m \notin [-1,\infty) \times Q$ and hence as $\psi(m) = \psi_s(m)$ for all $s \geq n$, the equation $\psi(m) = l$ holds which finishes the proof showing surjectivity of the map $\psi : M_0 \to M$.

Additionally, the theorem claims that ψ can be constructed to equal the identity in a chosen neighborhood U of $[0,\infty) \times Q$. This can be obtained in modifying the immersion φ such that $[-1,\infty) \times Q$ is contained in U. Doing this in shrinking each ray condition (1) will still hold. Then the constructed function g_s can be cut off to zero on U^c such that all vector fields constructed on M vanish on U^c. By force

$$\psi|_{U^c} = \mathrm{id}_{U^c}$$

□

A few examples may be given both for which the result holds and to which the theorem does not apply.

Let $N = [0,\infty) \times Q \subset M$ denote the local submanifold which is excised from M. If N is 1-dimensional the result holds, more generally it holds for N being isotropic. On the other extreme, for N being symplectic the theorem does not apply. One may also construct straightforward examples where the rank of ω on N jumps from point to point. For this, let M_1 and M_2 be symplectic manifolds, $N_1 \subset M_1$ be isotropic and of the form $[0,\infty) \times Q_1$. Then $N_2 \subset M_2$ can be chosen to be an arbitrary closed submanifold. The result holds for $N_1 \times N_2 \subset M_1 \times M_2$ and N_2 can be chosen such that the rank of the 2-form on $N_1 \times N_2$ jumps.

If M is of dimension 4, then if N is of dimension 2, N must be Lagrangian while the case in which N is of dimension 3 seems to be difficult to describe.

Applying this to $\mathbb{R}^{2n}$ with standard $\omega = \sum_{i=1}^{n} \mathrm{d}x_{2i-1} \wedge \mathrm{d}x_{2i}$ the result holds to the case $N = \{(x_1, \dots, x_{2n}) \in \mathbb{R}^{2n} \mid x_1 \geq 0, x_2 = 0\}$ but the theorem does not apply to all cases $N = \{(x_1, x_2) \in \mathbb{R}^2 \mid x_1 \geq 0\} \times N_2$ for $N_2 \subset \mathbb{R}^{2n-2}$ being a closed submanifold or any open subset $N_2 \subset \mathbb{R}^{2n-2}$. Furthermore the theorem treats the cases

$$N = \left\{ (x_1, \dots, x_{2n}) \in \mathbb{R}^{2n} \;\middle|\; x_1 = 0, \sum_{i=2}^{2n} a_i x_i \geq 0 \right\}$$

if $a_2 \neq 0$. This provides few examples in the case of $\mathbb{R}^4$ with $\dim N = 3$.

In [Str20], the author proves that a manifold with exact symplectic form ω admits a nowhere vanishing primitive β to ω, i.e. $\omega = \mathrm{d}\beta$, a property required in the paper of Blohmann and Weinstein ([BW18]). The Theorem simplifies the last step of the proof in [Str20] where the nowhere vanishing primitive is constructed from a primitive with isolated zeroes inductively. This can be done with this paper's Theorem in one step by choosing rays covering the zeros and then apply the global symplectomorphism of the complement of the rays to the initial manifold.

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