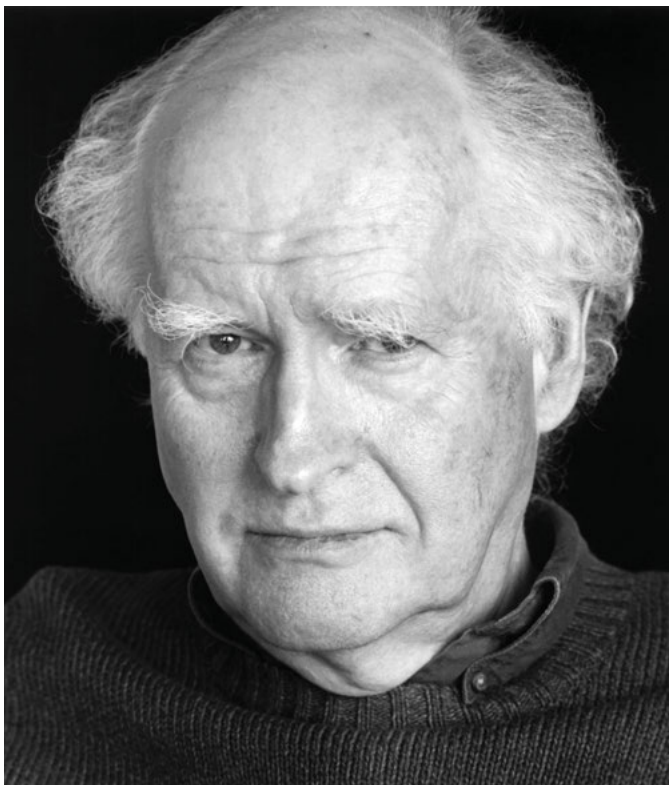


DEDICATION



John N. Mather

This is the special issue in memory of Professor John N. Mather, Part III.

PREFACE

The current volume is out of the International Conference at Sanya on Singularity theory and Dynamical Systems – in memory of John Mather in December 11-15, 2017, though the original scheme was to celebrate his 75th birthday.

John has made original and foundational contributions to mathematics, especially to singularity theory and Hamiltonian dynamical systems. While John was a graduate student he initiated the work of singularity theory. His series of papers on the subject set up the foundations and the papers became classical. A number of articles on singularity theory in this volume continues the work in this spirit. John also spent much of his time to set up the foundations for Hamiltonian dynamical systems. With this foundation people can make many deep applications. There are a few papers on this in the volume. It serves both as a useful introduction and an excellent access to the current research. There are papers related to John's other work on foliations and characteristic classes. We are indebted to all participants for their contributions.

We are grateful to the Mather family for their contributions to the conference and to the volume. Naomi participated in the conference. She also provided special family photos and John's brief yet most valuable self introduction to his mathematical life. We are also indebted to Peggy for her essay sharing with us the vivid life of John. Mara's video also moved us in the conference.

We would like to thank the Tsinghua Sanya International Mathematical Forum for their support and assistances for the conference. It left us all with very pleasant memories. We would like to thank the International Press for their patience and assistance for the preparation of this memorial volume.

The Guest Editors,

Sen Hu, University of Science and Technology, China;
Stanisław Janeczko, Polish Academy of Sciences, Poland;
Stephen S.-T. Yau, Tsinghua University, China;
Huaiqing Zuo, Tsinghua University, China.

PERSONAL RECOLLECTIONS

Photos



John's grandfather, Wiley Mather, as a child in the carriage, and great-grandfather (in top hat) in Lincoln, NY, 1889.



John's two sets of grandparents. From left to right: Thomas John Clark, Minerva Clark, Glenn Mather, and Wiley Mather.



John's parents, Mary Adele and Norman Mather, during WW II while Norman was teaching the new technology of radar to Princeton University EE undergraduates under the auspices of the US Navy.



John's parents.



John and his parents.



John's childhood.



John “assisting” his sister, Peggy Mather, with a very rare treat.



Left: Peggy, Right: John.



John and his mother, Mary Mather.



John studying the board while in high school.



John standing in front of the College Road row house in his high school years.



John and his wife, Naomi Mather, holding their newborn daughter, Mara Mather.



John and his daughter Mara.



John and his children.

Upper row, from left to right: Emily Mather, Frank Mather, Thomas John Mather, and Mara Mather.
John and Naomi Mather are below.

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John Recounts the Beginnings of his Interest in Mathematics

I remember being fascinated by logarithms when I was six years old. My father must have explained them to me. He was a professor of electrical engineering at Princeton. From time to time, he liked to teach me some aspects of elementary mathematics.

When I was eleven or so, I found one of my father's engineering textbooks on a shelf at home and I spent a considerable amount of time delving into it. It was basically a calculus text, including the calculus of variations, motivated by lots of applications to engineering problems. There was a lot that I did not understand. I am sure that I could not have passed an exam on the subject of the book. Nonetheless, I found it very interesting, perhaps the more so because I did not understand it very well. I found the fact that one could solve "real life" problems (such as finding the shape of the cable of a suspension bridge) by simple paper- and-pencil calculations very appealing.

My interest in mathematics led me to buy various math books from Dover Publications and the Princeton University bookstore when I was in high school. I bought and delved into Solomon Lefschetz's *Topology*, the first volume of Claude Chevalley's *Lie Groups*, books on group theory by R. D. Carmichael and William Burnside, an English translation of a three-volume *Cours D'Analyse* by Edouard Goursat, and Paul Halmos's *Finite Dimensional Vector Spaces*. I remember spending a great deal of time with all of these books because they fascinated me. On the other hand, with the exception of Halmos's book, which I studied thoroughly, what I understood was only a fraction of what these authors had to say.

In my last semester in high school, I was fortunate to be able to take part in a program initiated by Professor Albert Tucker of the Princeton mathematics department. This enabled bright high school students to take math classes at Princeton University. I was the first to do so. After a brief oral examination, I was

placed in a junior-level course in abstract algebra, taught by Ralph Fox. I did perfect work in the course.

I took my undergraduate degree at Harvard University and my graduate degree at Princeton University. When I was in my first year at Princeton, I read notes by Harold Levine of lectures by Rene Thom on singularities of mappings. I soon solved several open problems described in the notes. My solutions of these problems appeared in a series of six papers on smooth stability of mappings. After completing my degree at Princeton, I spent two years at Rene Thom's institute, Institut des Hautes Etudes Scientifiques (IHES) in France. Thom was very accessible and I was very glad that I got to know him. Thom thought he had proved the density of topologically stable mappings, but he had not convinced other mathematicians. I found a modification of his method that mathematicians accepted as a proof. This proof relied mostly on ideas of Thom but also used one significant innovation of mine.

I spent the next four years at Harvard. When I was there, I worked mostly on the theory of foliations and Andre Haefliger's classifying space. Bill Thurston found a brilliant generalization of my results, leading to the Mather-Thurston theorem, which reduces the study of the topology of Haefliger's classifying space to questions in the homology of groups of diffeomorphisms, most of which, however, remain open.

After four years at Harvard, I decided that I liked Princeton better. I have been at Princeton ever since.

One day, I heard Ian Percival speak at Princeton. He introduced an unusual sort of Lagrangian in his talk. A couple of years later, I made a serendipitous discovery that his Lagrangian could be used to prove an existence theorem. (A similar theorem was proved independently by S. Aubry and P. Y. LeDaeron.) This existence theorem proved to be of considerable interest to experts in Hamilton dynamics, and I have been working in related topics ever since.

John Mather Our Ancestry and Childhood Together

by Peggy Mather

John Mather's life was characterized by a passion for scholarship: particularly in mathematics, but also through a life-long, carefully constructed accrual of world historical knowledge. He had a deep regard for truth and integrity and a great, though tempered, impatience with scrambled thinking. At his memorial service, so beautifully arranged by Princeton University, he was described by his colleagues as being remembered for what his family also knew him to be: a kind man.

Naomi Mather, John's thoughtful wife since his years of teaching at Harvard, has asked me, John's younger sister by five years, to relate some observations about John's and my family, and some memories of our years together at home with our parents. It is a pleasure to do so.

Going back in time, John's and my Mather family forebears arrived here in North America in 1635, well before the country was founded. Richard Mather, who crossed the Atlantic Ocean from England in the 1600s with his family, was a Puritan, an observer of a severe religious orthodoxy. Thankfully, that extreme orthodoxy quickly died out. Over the centuries since their arrival in Boston and the Massachusetts Bay Colony, and their contributions in helping influence major institutions such as Harvard University, many of the Mather men segued into the Methodist ministry down to the latter part of the 1800s when our great-grandfather, Franklin Mather, was the last to join the ministry. That man's son, our grandfather, broke from the family tradition and turned his attention toward secular scholarship and formal teaching, as our own father did, and, of course, as John did himself.

Apparently a mather, in medieval England, was a cutter or harvester of hay. In the 21st Century, this humble derivation of the family name is somewhat amusing. Also amusing, is that our John, a mathematician's mathematician, didn't descend from a "math - er" (what one might imagine to be a person engaged with math). His surname, misinterpreted in this way, does however, seem particularly appropriate. Asked once if his colleagues had ever remarked about the poetic irony of his surname, John responded to me with his sometimes startling succinctness: "No." (i.e., "This subject doesn't interest me. Let's move on.") John could convey a lot with just one word.

John's paternal grandparents were exceptional people. Our grandfather, Wiley Wells Mather, was a man of great intellect, a seeker of explanations, historically speaking. Early in his career, he taught high-school level American History, moved on to teaching at the junior college level, and late in life became an estate attorney, to help set up and settle the estates of what seemed like an amazing number of friends and acquaintances. Like his ancestors, Wiley Mather had a need to explore the well-springs of Christianity. He became a self-taught biblical scholar. As a young man he possessed a near photographic memory and was linguistically gifted. He learned Greek and some Sanskrit in order to interpret original texts. He particularly relished discovering mistakes in the translations of early manuscripts, of which there were quite a few, and he studied the impact of these mistakes, which could be profound.

The main room in Wiley's house was lined with floor-to-ceiling bookcases. He frequently consulted volumes high above floor level with the use of an ugly, clanking aluminum ladder, surprising his young grandchildren visiting their California grand-

parents from the East Coast. His physical agility, and his sharp recall, evidenced by his having memorized the exact location of just what he was looking for to support his detailed exposition or argument, was impressive to us children. He rarely stopped talking, something Wiley's son, our father, wryly remarked about on occasion. He was viewed by his grandchildren, on the other hand, as being delightfully, animatedly, entertaining, and so admirably knowledgeable. I once gave John a small book by Richard Feynman, "Surely your Joking, Mr. Feynman!" about his years at Princeton. John found it to be hilarious, in the same way that our very clever grandfather had struck us during our rare visits to Upland, California from Princeton, New Jersey, more than 3,000 miles away... unfortunately.

John's and my Mather grandmother, Glenn Mather (nee Shaw) was a home-grown scholar herself. Her passion was human nutrition. She had her own extensive library, and fashioned the nutrition in the Mather household according to the best information available at the time: a time when nutritional science was just beginning to be respected and published. She carefully shopped for and cooked three meals a day, each of which contained dishes from the so-called five main food groups. She labored at this for many decades, which was a very laborious undertaking, to say the least. She cultivated a kitchen garden, and learned to tinker with pipe wrenches and other tools on the irrigation system, to repair and optimize it, all the while wearing her high heels, a lady of her day. Her meals, simple creative masterpieces, kept the two of them healthy, clear minded and vigorous well into advanced age. Wiley lived to be 101, unusual at that time, and Glenn, his wife, lived into her 90s. No French fries, and certainly no manufactured food ever appeared on their expansive dining room table. They drank raw, unpasteurized milk from a closely-monitored and licensed dairy, and paired each meal with carefully researched nutritional supplements served up in shot glasses (which never held liquids) next to beautiful china and silverware. Glenn Mather approached feeding her family as a very significant endeavor and strived to make it a scientific undertaking, adjusting here, augmenting there as available research evolved. Fresh, fresh, fresh was her mantra. Agricultural pesticides and fertilizers were closely monitored and avoided. They lived through the time of heavy DDT usage in the United States, until it was banned, without its presence in their food. Some of her literature, "Vital Foods for Total Health," pamphlets on amino acids, the benefits of juicing, and the wisdom of stainless steel cookware, reside to this day on my own bookshelves. I've framed one of her pipe wrenches against the cover of one of her nutrition books as a personal nod of respect for her amazing, unflagging labor and search for expanding knowledge. Thankfully, after he retired from teaching, Wiley joined in with cleaning up after these hundreds, upon hundreds of "A+ in nutrition" meals. One decade equalled nearly 11,000 meals. Glenn and Wiley were married for more than seven decades. And I'm not sure Wiley ever retired as an attorney, but he was probably home like clockwork for every meal, retired or not. The two of them were awe inspiring.

John's father, Norman Mather, was their eldest child. Norman Mather absorbed his parents' respect for discipline and love of learning. As a child he submitted to the rigors of many years of piano lessons and ultimately became a very good amateur pianist. While John was growing up, in the small households our parents could afford, and later in the house they built, the sounds of many composers, but especially Beethoven, Schumann, and Chopin, filled the atmosphere and drove everyone to their respective, personal "quiet spaces" (our bedrooms) so we could concentrate, hear ourselves think, and carry on conversations with each other like fugitives. John spent

many hours in his bedroom studying while piano music filled the house and very subtly vibrated the walls every evening. In his 70s, John once remarked to me that he had enjoyed that music a great deal and missed its presence. Many people have observed that scientists and mathematicians frequently love music, Bach being a favorite; but Chopin, whose music was much loved by our father, was the most frequently played composer in our household. Chopin's minor keys contributed to an atmosphere of slight apprehension, as though we were being warned in a... well, a minor way. But then Chopin would revert to a major key, and all would be well... ah, resolution! One Christmas I gave our father recordings of Chopin himself playing his own stunning compositions, fascinating to anyone trying to reproduce his music technically and emotionally, as our father had done for years. Chopin playing his own piano was recorded in Thomas Edison's very first experiments to capture sound on wax cylinders in the 1800s, then "rediscovered" and transferred to tape more than a century later! I wish John could have seen his father's delight at receiving that gift, but John was already out in the world and no longer living at home.

As a graduate of University of California at Berkeley, Norman earned a BS in Electrical Engineering. Several years after he married our mother, and early in his career, Norman recognized, as was dawning on the nation as a whole, that the United States was heading towards an inevitable entrance into World War II as a major combatant to help save Western civilization from fascism. Through mentors and voluntary enlistment before a nation-wide emergency draft was imposed, he was able to guide the direction of his future toward involvement with the U.S. Navy's radar program. Ultimately, after intensive training at MIT through the auspices of the U.S. Navy, he was assigned by the Navy to teach this new technology to undergraduates at Princeton University. I remember, as a young child, visits to see Professor Willis, most probably one of our father's promoters and mentors, and John must have accompanied our parents many times in their visits to honor Professor Willis in the years before I was born. Our family has photographs of Norman in the uniform of a Navy officer, teaching the rudiments of radar to Princeton electrical engineering undergraduates peering into their oscilloscopes during the Nation's years as an active, WWII combatant.

John's father was exceptionally good at teaching (a talent he wasn't fully aware of prior to his years in the Navy) and he was strongly encouraged to apply for a faculty position at Princeton. After the war ended, he also earned an MS at Princeton so as to enhance his prospects. He succeeded with his goal of being hired by Princeton. For the remainder of his career, after achieving the rank of full professor, he was one of very few, if not the only full professor at Princeton who did not have a PhD! While John was still a child, Norman became Assistant Director of secret Project Matterhorn, which later segued into Princeton's Plasma Physics Laboratory. He designed and helped build much of the original equipment.

Immediately following the War, John was the child of a young, new Princeton faculty member and his wife. I was not yet born. Perhaps our father's position was a high-prestige job, but it wasn't handsomely paid. Our parents had to impose strict discipline on their finances and make hard choices. But John, late to acquire language*, which greatly concerned his parents, was exceptionally good at understanding mathematical concepts; so good, they didn't have to expend resources to help develop his startling and unusual talent, thanks to his father's teaching abilities. John had a voracious curiosity and spent most of his time trying to figure out how to make sense of the world by asking questions without needing to be encouraged. In

fact our mother, worrying that he was becoming too one-sided in his primary interest, numbers, tried to encourage him to go outside and play with all the other kids yelling and laughing, competing and occasionally fighting as they devised games for themselves and struggled to win contests of physical prowess. He didn't want to: playing didn't interest him. She did succeed in encouraging John to join Cub Scouts. John and I were both taught how to ride a bike, how to swim, etc., and took various lessons in this and that as time went by. We both enjoyed mother-organized birthday parties. But years later, when she signed him and me up for tennis lessons, he balked. As he explained to me, physicality was not his thing, and early on he had recognized that tennis was a game encouraged in a certain stratum of society as an unspoken mark of class distinction. This seemed ridiculous to him, and he refused to be part of it. I, as his much younger sibling, not fully understanding his arguments, nevertheless agreed with him 100%! United against tennis! Our mother relented.

John was unusually lucky to have had a father who recognized his son's innate mathematical ability, and the two of them spent time on occasion, after our father came home from a demanding day of teaching and studying for his MS, going over rudimentary mathematical concepts. John was exploring logarithms at the dining room table when he was in the first grade and not yet tall enough to plant his elbows on the table, nor his feet on the floor. I, on the other hand, was crawling around underneath the table, while my mother was cleaning up after dinner in the kitchen, pulling at my father's shoelaces and trying to grab my brother's dangling, swinging feet, an annoyance and not charming while you're trying to concentrate on logarithms at age six. Norman once remarked that when John was very young, he, Norman, would bring his sit-down sessions with John to a quick halt as soon as John's interest started to flag. John was never pressured to soldier on and try to go a bit further. Our master teacher father did something wonderful: he planted a seed, or perhaps simply watered a tiny seedling: he helped John develop a respect, if not a reverence for the beauty of math and science. He coached his son into a life of intellectual inquiry grounded on real and conceptualized order together with an enthusiasm for an observance of patterns, and an appreciation for the breath-taking elegance of the natural world, all described in numbers.

Our maternal great-grandparents were Scots whose families had immigrated to Canada in the nineteenth century. Our grandfather, Thomas John Clark, was a man of great character and industry. When Thomas was a nineteen-year-old, his parents had both died prematurely, and Thomas became the sole support of his many younger brothers and sisters. This extremely heavy and demanding responsibility (something impossible to meet in today's world) prevented him from going to college, but he managed to keep his siblings together until they each reached the age of majority and had the ability to become independent. He pursued a lifelong interest in literature, particularly American literature: Emerson was a strong favorite. He and most of his siblings eventually moved to the United States and Thomas ultimately ended up with his own small family in Berkeley, California. He was a poetry lover and writer, and memorized many long poems which he would recite at appropriate moments, like walking the beautiful western edge of our broad continent with his young daughter Mary Adele, our mother, against a visual and auditory backdrop of crashing waves. Despite the huge challenges he had faced as a very young man, he was a gentle, loving person, adored by his children. John's elder son, Thomas John, known as TJ, was named after his great-grandfather.

John's maternal grandmother, Minerva Clark (nee Heppenstall) was a stern per-

son, very discipline-oriented, and somewhat intimidating. At a time when most clothing was homemade or made by professional tailors and seamstresses, she was an accomplished seamstress herself, and designed and painstakingly sewed most of the clothing for herself and her family. She designed and pieced together simple, but starkly beautiful quilts, an American folk tradition. Though untrained formally beyond high school, she had a passion for architecture and designed several houses of increasing size still standing proudly in the hills of Berkeley, California. She had an innate talent for design and form. As a woman, it's delightful to seek out those graceful houses in the Berkeley Hills and be able to say, "My grandmother designed that house!"

Thomas and Minerva Clark had three children, the youngest of whom was Mary Adele, our mother. Mary Adele took after her very warm and loving father, was an ebullient person, infectiously enthusiastic, and very gregarious. When she was a tiny girl, her father nicknamed her "Little Miss Mary Sunshine," a satisfyingly rhythmic name, and very apt. When Mary Adele was a teenager, she babysat for several years for a very young girl across the street named Portia. Like Shakespeare's Portia, this tiny neighbor was the daughter of a wealthy man. Sadly, she was a sickly child and died quite young. But Portia's parents were so impressed with Mary Adele and her very positive influence on their beloved daughter, that at the time of our parents' marriage, they gave the newlyweds a wonderful gift: sterling silver flatware with many multi-piece place settings, together with every imaginable complementary sterling table-top implement to support it during holiday celebrations. This was extraordinary because the Great Depression had taken a great toll on the Nation, and it was yet a very, very difficult time for many Americans, especially newlyweds just starting out and striving to establish themselves. That weighty load of silverware represented a small fortune and was a lavish gift. It was used every day (sterling doesn't wear out) by the young Mather family and to John and his impressionable younger sister, it became emblematic of others' appreciation of our mother's wonderful disposition. In a male-centric culture, our mother had influence! And for the very best reasons.

Mary Adele and Norman met as undergraduates at the University of California at Berkeley. Mary Adele was an economics major, and by her own description, barely squeaked through to graduation. But she had a well developed attribute that served her over and over again throughout her life: what she called common sense. She had a natural talent for identifying and weighing options and deciding in which direction to take things when attempting to solve problems. In today's parlance, she was a pragmatist, in the positive sense of the word.

And she was an exceptionally good mother. She guided John in ways that were designed to broaden his horizons, particularly socially. She enrolled him in Cub Scouts, gently suggested he stop studying and go outside and play with that great demographic bulge of children in the neighborhood that was preceded by John and his contemporaries, the War Babies, and hugely enlarged by the Baby Boomers, born post-1945 (John was born in 1942). Oddly, the great majority of these children were boys.

When he was about eight years old, our mother encouraged John to make his little three-year-old sister a Christmas present: a doll house constructed using a wooden orange crate, long and narrow with two internal divisions, which he upended, cutting holes for windows. It had three "floors" like our little home. He papered the "walls" with real wallpaper, and laid real linoleum on the tiny kitchen floor with scraps left over from actual household renovations. This may have been the only time

John labored to make a real, three-dimensional object. John was taught and carefully encouraged to look out for his younger sister, to read to me and to encourage me. He developed that heightened sense of responsibility that many first-born children are encouraged to cultivate toward their younger siblings. He extended that concern and generosity into adulthood and made a major, positive impact on my life.

Since John's father was a faculty member, he and his family were entitled to live in University- owned faculty housing for below "market rate" rent for something comparable in the community at large. Their first home was a tiny unit, two stories at and above ground level and a basement replete with a coal-burning furnace which sat next to a work bench that frequently had a couple of oscilloscopes resting atop it, scopes our father was repairing. A pile of zero-tech coal, a planted shovel sticking out, and 40s-era high-tech oscilloscopes, sat side by side.

Our tiny apartment at 36 Edwards Place was part of what's known of, as a "row house." Many young families living between adjoining side walls filled that row house to near bursting. It seemed that every young couple was starting, or adding to their family. Edwards Place is a dead-end street, directly across from the Princeton University Store, and during the summer months of no school (yay!) the children could play in the street without danger from through-traffic. The raucous noise of yelling kids began early and didn't let up until dinnertime. All the neighbors were professors, their wives, and their children, all housed in that regretfully cramped row house. Across the street other University staff and their families were housed. Dutch Schoch took particular delight in all the kids, encouraging success over physical challenges. He was Princeton's heavyweight crew coach. He died prematurely, and later as a teenager, I accompanied my mother on a visit to his wife's beautiful home (not on Edwards Place). At some point, many Princeton trees contracted Dutch elm disease. Mrs. Schoch got a cross section of one of the elms felled intentionally by the University because Dutch elm disease was contagious. A play on words? She made a coffee table, perhaps five feet in diameter, out of that cross section. The tree was so old and huge it may have started growing well before Anglos ever set foot in what was to become colonial Princeton. Angus McBride, another staff member's teenager, sometimes put on his gorgeous kilt in the McBride tartan plaid, and his Scottish, tasseled high socks and other gear, and would march up and down the street playing his bagpipes, while we kids marched behind our very own pied piper, stepping slowly to that irresistible Scottish wail. Ours was a fairly tightly-knit community with stay-at-home mothers (this was the mid-1940s into the 1950s) sharing child rearing views, domestic problem-solving ideas and short-lived time off from the omnipresent drudgery of endless household chores. There were very few household appliances back then, and housework abounded. The fathers discussed serious matters when they had time to relax on summer evenings outside, chatting with each other. There was a general atmosphere of relief and optimism, now that the War was over. A couple of the professors on Edwards Place went on to win Nobel prizes, and our parents befriended many whose entire careers involved scholarship at Princeton, as did Norman's. Close friend Herbert Bailey, became Director of the Princeton University Press. Lawrence Thompson became Robert Frost's official biographer. Ernestina Parravano, wife of Professor Guiseppe Parravano, became a beloved "hostess" for University of Michigan (Ann Arbor) faculty wives from foreign cultures. She herself had been a "foreign wife" in the Princeton faculty community. All the children went to the community's public schools when old enough for Nursery School, and then Kindergarten, and continued all the way through high school. The public schools were excellent. Exceptional teachers

made all the difference.

At age eleven, while still living on Edwards Place in that tiny, noisy, bulging row house, John, bored with being urged to play outside with the other kids his age so often, found something much more interesting: one of his father's engineering textbooks residing on our parents' bookshelves. In his own words, "I spent a considerable amount of time delving into it. It was basically a calculus text, including the calculus of variations, motivated by lots of applications to engineering problems. There was a lot that I did not understand. I am sure that I could not have passed an exam on the subject of the book. Nonetheless, I found it very interesting, perhaps the more so because I did not understand it very well. I found the fact that one could solve 'real life' problems (such as finding the shape of the cable of a suspension bridge) by simple paper-and-pencil calculations very appealing."**

Also on those bookshelves was an encyclopedia of human health and diseases with gruesome black and white photos of people suffering from various horrible ailments, a black bar superimposed across their eyes to protect their identities. I know what full-blown elephantiasis and grotesque goiters look like... did John? Most likely. Perhaps more significantly, a much loved, well thumbed, and beautifully illustrated copy of Bertrand Russell's "Wisdom of the West," resided on those bookshelves, which may have been the first book to pique John's life-long interest in history. I can still remember the smell of that book, at a time when printers' ink was pungent. It's simply understood illustrations were dynamite for a child looking for something, anything, to catch his or her interest on a dark, winter's weekend day. There wasn't much else to look at for us kids, except our schoolbooks and books we acquired on our own. The New York Times and the New Republic were too difficult for me to understand, although perhaps John did, and 25-cent comic books, something I, but not John, coveted, were discouraged with disdain. Television watching, very new at the time, was strictly limited by our parents. The CBS News with Walter Cronkite, was acceptable, but little else. We were encouraged to go to the Public Library, located in a large colonial house on Nassau Street at that time, and had the freedom to walk there alone, even as young children proudly clutching our library cards, which seems amazing in this day and age. Frustrated with so much reading, and after much whining, I was finally allowed to watch "Disneyland" on TV on Sunday nights, but by that time, John had no tolerance for fantasy, and thought it was, well, ridiculous and dumb. Late in life, when John was invited to spend a year as a visiting scholar at the Institute for Advanced Study, his colleagues there screened the movie, "The Man Who Knew Infinity." John enjoyed it! He had consistently eschewed movies, television, and drama in general. Not long after he got sick, and during the two year physical denouement he experienced, I sent him a copy of the HBO miniseries "John Adams," a wonderful, extremely well done dramatic adaptation of David McCullough's Pulitzer Prize-winning biography, which John had so very much enjoyed reading. The discs were never opened, the album still in its cellophane wrapping to this day. He'd impoverished himself... a whole area of the arts was never enjoyed by him, perhaps in part because he'd concluded "Disneyland" was insipid.

When our father made full professor, our family was eligible to move to another, bigger and more spacious University-owned row house on College Road, across the street from McCarter Theater. It had a slate roof, and occasionally a piece of sharply-edged slate would come loose and plummet to the ground like a guillotine. Anyone standing in the wrong place could have been decapitated, but thankfully, no one was. Our 8 College Road apartment was so much bigger than our previous home. Our

father bought a Baldwin spinet piano and began his very loud, though admittedly quite skilled, nightly concerts. When John was a young student at Princeton High School, occasionally his friends would come over and they would play chess. I think they all may have been members of PHS's Chess Club. Out of respect for their need to concentrate, Norman's piano would remain silent while they were there. John's friends at school were the smartest boys in his class and several enjoyed notable careers. Peter Kann, who much later became publisher of "The Wall Street Journal," came over to play blind-man's chess with John, in which one player would have to memorize the positions of all the pieces on the chessboard, together with potential moves, and then actual moves, while the other player had sight of the board and physically moved the pieces. An extraordinary exercise in memory and concentration indeed... most especially for the "blind" player, of course, but they changed sides, and the blind became the sighted.

John's free time was spent almost entirely on study, completely self-directed study, completely self-chosen study. In his own words again:

"My interest in mathematics led me to buy various math books from Dover Publications at the Princeton University bookstore when I was in high school. I bought and delved into Solomon Lefschetz's *Topology*, the first volume of Claude Chevalley's *Lie Groups*, books on group theory by R.D. Carmichael and William Burnside, and an English translation of three-volume *Cours D'Analyse* by Edouard Goursat, and Paul Halmos's *Finite Dimensional Vector Spaces*. I remember spending a great deal of time with all of these books because they fascinated me. On the other hand, with the exception of Halmos's book, which I studied thoroughly, what I understood was only a fraction of what these authors had to say."**

While a senior in high school, John was invited to take part in a program initiated by Professor Albert Tucker, a friend of the family and a math professor. This enabled bright high school students to take math classes at Princeton University. John was the first to do so. After a brief oral examination, he was placed in a junior-level course in abstract algebra, taught by Ralph Fox. He did perfect work in the course.

After graduating from high school, John left for college. My very brief description of the time John and I shared together living in our parents' household ends here. We were fortunate children, carefully raised by devoted and educated parents, with the advantages of a community known for people and resources committed to encouraging young, inquiring minds.

My brother John was a brilliant mathematician, but perhaps more significantly, a good man, a kind and generous man. He revered the power of the mind, but didn't hold that regard he had for the mind over others with less developed acuties. His absence is felt daily by his family.

* It might be interesting to some to learn (according to our mother's reminiscences of John at about age three) when he first started to speak, John would do the following. If a word ended in a hard consonant sound, he would pronounce that hard consonant sound at the beginning of the word, not at the end. E.g., "dog" was pronounced something like "god," and "hat" as "tah." I have wondered if his very early efforts to reproduce language, and his first stumbling attempts, reversing the order and placement of consonants, ever had any reflection in his ability to untangle mathematical concepts, and, in turn, construct unique responses.

** John's quote about "the Beginnings of his Interest in Mathematics" is in the following book: Cook, Mariana. *Mathematicians*. Princeton, NJ: Princeton University Press, 2009.