METHODS AND APPLICATIONS OF ANALYSIS.
Vol. 28, No. 3, pp. 249–264, September 2021

HSIAO'S PDE THEORY ON SEMI-CONDUCTOR AND PLASMA AND THEIR APPLICATIONS*

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Dedicated to Professor Ling Hsiao on the occasion of her 80th birthday

Abstract. Appreciating of the contributions on PDE of Professor Ling Hsiao, we review Hsiao's PDE theory on semiconductor and plasma and their applications. As an example of application of Hsiao's PDE theory on semiconductor and plasma, we study the asymptotic regimes of the compressible Euler-Maxwell equations on plasma. The derivation of the one fluid non-isentropic Euler-Maxwell system from the two-fluid model in terms of the asymptotic expansion method is performed, and the mathematical analysis of the stability of both constant equilibrium solutions and non-constant equilibrium solutions is given on $\mathbb{T}^3$ or $\mathbb{R}^3$.

Key words. Hsiao's PDE theory on semiconductor and plasma, Drift-diffusion equations, Euler-Maxwell equations, small parameter limits, stability.

Mathematics Subject Classification. 35B40, 35Q35, 35Q60.

1. The hydrodynamic models of Hsiao's PDE theory on Semiconductor and plasma. The hydrodynamic models involved in Hsiao's PDE theory on semiconductor and plasma describing the electron flow are given by [57, 41, 39]

$$\begin{cases} n_t + \nabla \cdot (nu) = 0, \\ u_t + (u \cdot \nabla)u + \dfrac{1}{n}\nabla p(n) = \nabla \Phi - \dfrac{u}{\tau} \\ \Delta \Phi = n - b(x), \quad \Phi \to 0, \quad \text{as } |x| \to +\infty, \end{cases} \tag{1.1}$$

for $(x,t) \in \mathbb{R}^d \times [0,+\infty)$, $d = 1,2,3$, where n, u, Φ denote the electron density, electron velocity and the electrostatic potential, respectively. The constant $\tau > 0$ is the velocity relaxation time, the function $b(x)$ denotes the prescribed density of positive charged background ions (doping profile), and the pressure-density function $p = p(n)$ has the property that $n^2 p'(n)$ is strictly increasing function from $\mathbb{R}^+$ onto itself. A usual hypothesis is

$$p(n) = a^2 n^\gamma, \quad n > 0, \quad a \neq 0, \quad \gamma \geq 1. \tag{1.2}$$

System (1.1)-(1.2) is a simplified multi-Dimensional (m-D) hydrodynamic model which was analyzed by Degond and Markowich [13] for the first time in the stationary case. For the 1D case, the Cauchy problem and the initial-boundary value problem of (1.1) has been extensively studied by many authors. In the stationary case, Degond-Markowich [12] proved the existence and uniqueness of steady-state solutions in the subsonic case. Gamba [24] discussed the existence and uniqueness of steady-state solutions in the transonic case. In the dynamic case, Zhang [91] and

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Marcati-Natalini [59] investigated the global existence of weak solutions of the 1D initial-boundary value problem and the Cauchy problem, respectively, by using the tools of compensated compactness. The corresponding results on the zero relaxation limit have been obtained also in [42, 43, 59]. Luo et al. [57] and Hsiao and Yang [41] investigated the asymptotic behavior of smooth solutions of the Cauchy problem and the initial-boundary value problem of (1.1), respectively, and proved that under appropriate conditions on the doping function $b(x)$ and the pressure function $p(n)$ the corresponding steady-state solutions of the simplified hydrodynamic model and the drift-diffusion model are exponentially asymptotically stable.

For the m-D case, besides the local classical solutions obtained in [46, 58], only steady-state solutions in the subsonic case and in the dynamic case solutions with geometrical structure (symmetry) or without vorticity were studied in [9, 13, 17, 29, 37, 40]. Chen and Wang [9] established the existence of global weak solutions with geometrical structure of system (1.1). Hsiao and Wang [40] considered the smooth solutions of the system and established the global existence and asymptotic behavior of the spherically symmetrical solution of (1.1) with $\gamma = 1$. Engelberg et al. [17] also studied the critical threshold phenomena of 1D and m-D pressureless Euler-Poisson equations with geometrical symmetry in the m-D case and with and without relaxation. Guo [29] investigated the irrotational Euler-Poisson equation system without relaxation and without geometrical symmetry and demonstrated that the smooth, irrotational initial data which are small perturbations of a fluid at rest lead to globally smooth, irrotational solutions of the Euler-Poisson system. Hsiao, Markowich and Wang [39] studied the asymptotic behavior of globally smooth solutions of the Cauchy problem for the m-D system (1.1), they proved that smooth solutions (close to equilibrium) of the problem converge to a stationary solution exponentially fast as $t \to \infty$, and obtain

Theorem 1.1 (see [39]). *Assume that $b(x) = B > 0$ and (1.2) holds. Assume that $n(\cdot,0) - B \in H^3(\mathbb{R}^d)$, $u(\cdot,0) \in H^3(\mathbb{R}^d)$, and $\nabla\Phi(\cdot,0) \in H^3(\mathbb{R}^d)$. Then there exist positive constants δ_0, depending only on B, such that if $\|n(\cdot,0) - B, u(\cdot,0), \nabla\Phi(\cdot,0)\|_{H^3(\mathbb{R}^d)} + \|(n_t, u_t, \nabla\Phi_t)(\cdot,0)\|_{H^2(\mathbb{R}^d)} \le \delta_0$, then there exists a unique global smooth solution $(n, u, \nabla\Phi)$ to the Cauchy problem of (1.1). Moreover,*

$$\begin{aligned}&\|(n(\cdot,t) - B, u(\cdot,t), \nabla\Phi(\cdot,t))\|^2_{H^3(\mathbb{R}^d)} + \|(n_t, u_t, \nabla\Phi_t)(\cdot,t)\|^2_{H^2(\mathbb{R}^d)}\\ &\le C_1 \left(\|(n(\cdot,0) - B, u(\cdot,0), \nabla\Phi(\cdot,0))\|^2_{H^3(\mathbb{R}^d)} + \|(n_t, u_t, \nabla\Phi_t)(\cdot,0)\|^2_{H^2(\mathbb{R}^d)}\right) e^{-\alpha_1 t},\end{aligned}$$

with positive constants α_1 and C_1.

Remark 1.1. *The parameter δ_0 which measures the allowed deviation from equilibrium, may depend upon the relaxation time τ.*

Furthermore, Li, Zhang and Zhang [49] considered the initial value problem for bipolar quantum hydrodynamic model for semiconductors

$$\begin{cases} \partial_t \rho_i + \nabla\cdot(\rho_i u_i) = 0, \\ \partial_t(\rho_i u_i) + \nabla\cdot(\rho_i u_i \otimes u_i) + \nabla P_i(\rho_i) = q_i \rho_i E + \dfrac{\varepsilon^2}{2}\rho_i \nabla\left(\dfrac{\Delta\sqrt{\rho_i}}{\sqrt{\rho_i}}\right) - \dfrac{\rho_i u_i}{\tau_i}, \\ \lambda^2 \nabla\cdot E = \rho_a - \rho_b - \mathcal{C}(x), \nabla\times E = 0, E(x) \to 0, |x| \to +\infty, \\ (\rho_i, u_i)(x,0) = (\rho_{i_0}, u_{i_0})(x). \end{cases} \tag{1.3}$$

Li, Zhang and Zhang [49] proved that the unique strong solution exists globally in time and tends to the asymptotical state with an algebraic rate as $t \to +\infty$. Namely

THEOREM 1.2 (see [49]). *Assume $\mathcal{C}(x) = c^*$ with c^* a positive constant, and $\rho_a^* > 0$, $\rho_b^* > 0$ are constants satisfying $\rho_a^* - \rho_b^* - c^* = 0$. Assume $P_a, P_b \in C^6$ and $P_a'(\rho_a^*), P'b(\rho_a^*) > 0$. Let the initial data satisfy $(\rho_{i_0} - \rho_i^*, u_{i_0}) \in H^6(\mathbb{R}^3) \times H^5(\mathbb{R}^3), i = a, b$, with $\Lambda_0 := \|(\rho_{i_0} - \rho_i^*, u_{i_0}) \in H^6(\mathbb{R}^3) \times H^5(\mathbb{R}^3)$. Then, there exists $\Lambda_1 > 0$ such that if $\Lambda_0 \le \Lambda_1$, the unique solution (ρ_i, u_i, E) of the IVP (1.3) with $_i > 0$ exists globally in time and satisfies for $i = a, b$ that*

$$(\rho_i - \rho_i^*, E) \in C^k\left(0, T; H^{6-2k}\left(\mathbb{R}^3\right)\right), \quad u_i \in C^k\left(0, T; H^{5-2k}\left(\mathbb{R}^3\right)\right), \quad for \quad k = 0, 1, 2.$$

Moreover, the solution (ρ_i, u_i, E) tends to $(\rho_i^*, 0, 0)$ at an algebraic time-decay rate

$$(1+t)^k \left\|D^k\left(\rho_i - \rho_i^*\right)\right\|^2 + (1+t)^5 \left\|\varepsilon D^6\left(\rho_i - \rho_i^*\right)\right\|^2 \le c\Lambda_0, \quad 0 \le k \le 5,$$

$$(1+t)^k \left\|D^k u_i\right\|^2 + (1+t)^k \left\|D^k E\right\|^2 + (1+t)^6 \left\|D^6 E\right\|^2 \le c\Lambda_0, \quad 0 \le k \le 5.$$

Based on the results obtained by Professor Hsiao's group, by using Hsiao's PDE theory on semiconductor and plasma, Guo and Strauss [31] proved the stability of semiconductor states of system (1.1) with relaxation terms under insulating and contact boundary conditions in $H^3(\mathbb{R}^3)$. By using an induction argument on the order of the derivatives of solutions in energy and time dissipation estimates, Peng [62] proved the stability of non-constant equilibrium states of system (1.1) in $H^s(\mathbb{T}^3)$ for $s \ge 3$. Recently, Li, Wang and Feng [52] investigated the stability of nonconstant steady-state solutions for 2-fluid non-isentropic Euler-Poisson equations in semiconductor.

The rest of this paper are arranged as follows. In section 2, we review the recent results about Drift-diffusion models of Hsiao's PDE theory on semiconductor and plasma investigated by Professor Hsiao's group. In section 3, we give some new results on Euler-Maxwell equations which are an example of applications of Hsiao's PDE theory on semiconductor and plasma.

2. Drift-diffusion models of Hsiao's PDE theory on semiconductor and plasma. The one-dimensional drift-diffusion models of Hsiao's PDE theory on semiconductor and plasma read (see [41, 84])

$$\begin{cases} n_t^\lambda = \left(n_x^\lambda - n^\lambda \Phi_x^\lambda\right)_x, \quad 0 < x < 1, \quad t > 0, \\ p_t^\lambda = \left(p_x^\lambda + p^\lambda \Phi_x^\lambda\right)_x, \quad 0 < x < 1, \quad t > 0, \\ \lambda^2 \Phi_{xx}^\lambda = n^\lambda - p^\lambda - D, \quad 0 < x < 1, \quad t > 0, \\ n_x^\lambda - n^\lambda \Phi_x^\lambda = p_x^\lambda + p^\lambda \Phi_x^\lambda = \Phi_x^\lambda = 0, \quad x = 0, 1, \quad t > 0, \\ n^\lambda(x, 0) = n_0^\lambda(x), \quad p^\lambda(x, 0) = p_0^\lambda(x), \quad 0 \le x \le 1. \end{cases} \tag{2.1}$$

The variables n^λ, p^λ, Φ^λ are the electron density, the hole density, and the electric potential, respectively. The constant λ is the scaled Debye length of the semiconductor device under consideration. $D = D(x)$ is the given function of space and models the doping profile (i.e., the preconcentration of electrons and holes). Because of the occurrence of p-n junctions in realistic semiconductor devices, the doping profile $D(x)$ typically changes its sign.

A necessary solvability condition for the Poisson equation in (2.1) subject to the Neumann boundary condition for the field in the fourth equation of (2.1) is global space charge neutrality

$$\int_0^1 \big(n^\lambda - p^\lambda - D\big)dx = 0.$$

Since the total numbers of electrons and holes are conserved, it is sufficient to require the following corresponding condition for the initial data :

$$\int_0^1 \big(n_0^\lambda - p_0^\lambda - D\big)dx = 0. \tag{2.2}$$

Usually semiconductor physics are concerned with large-scale structures with respect to the Debye length λ (λ takes small values, typically $\lambda^2 \sim 10^{-7}$). For such scales, the semiconductor is almost electrically neutral, i.e., there is no space charge separation or electric field. This is the so-called quasi-neutrality assumption of semiconductors or plasma physics, which was applied by Shockley [72] in the first theoretical studies of semiconductor devices in 1949, but was also applied in other contexts such as the modeling of plasmas [74] and ionic membranes [70]. Under the assumption of space charge neutrality, i.e., $\lambda = 0$, we formally arrive at the following quasi-neutral drift-diffusion model:

$$\begin{cases} n_t = (n_x + nE)_x, \\ p_t = (p_x - pE)_x, \\ 0 = n - p - D, \\ E = -\Phi_x. \end{cases} \tag{2.3}$$

This formal limit was obtained by Roosbroeck [69] in 1950. For general sign-changing doping profiles, the quasi-neutral limit (zero-Debye-length limit) is justified rigorously in the spatial mean square norm uniformly in time by Wang, Xin and Markowich with the help of multiple scaling matched asymptotic analysis in [84].

Theorem 2.1 (see [84]). *Let $l \le 1$. Assume the initial datum $(z_0^\lambda, E_0^\lambda)$ satisfies*

$$\begin{cases} z_0^\lambda = z_0^0(x) + \lambda f(x) z_+^1\left(\dfrac{x}{\lambda}, 0\right) + \lambda g(x) z_-^1\left(\dfrac{1-x}{\lambda}, 0\right) + \lambda z_{0R}^\lambda(x), \\ E_0^\lambda = E_0^0(x) + f(x) E_+^0\left(\dfrac{x}{\lambda}, 0\right) + g(x) E_-^0\left(\dfrac{1-x}{\lambda}, 0\right) + \lambda E_{0R}^\lambda(x), \end{cases} \tag{2.4}$$

with $E_0^0 \in C^{2(l+1)}[0,1]$,

$$E_0^0(x)|_{x=0,1} = -\frac{D_x(x)}{z_0^0(x)}|_{x=0,1}, \tag{2.5}$$

$$\big\|z_{0R}^\lambda(x)\big\|_{H^1} \le M\sqrt{\lambda}, \quad \big\|\partial_x^2 z_{0R}^\lambda(x)\big\|_{L_x^2} \le M\lambda^{-\frac{1}{2}},$$

$$\big\|\partial_x^j E_{0R}^\lambda(x)\big\|_{L_x^2} \le M\lambda^{\frac{1}{2}-j}, \quad j = 0,1,2.$$

Then, for any $T \in (0, T_0)$, there exist positive constants M and λ_0, $\lambda_0 \ll 1$ such that, for any $\lambda \in (0, \lambda_0]$,

$$\sup_{0\le t\le T}\Big(\big\|\big(z_R^\lambda, E_R^\lambda, z_{R,x}^\lambda, z_{R,t}^\lambda\big)\big\|_{L_x^2} + \lambda\big\|\big(E_R^\lambda, E_{R,x}^\lambda, E_{R,t}^\lambda\big)\big\|_{L_x^2}\Big) \le M\sqrt{\lambda^{1-\delta}},$$

for any δ with $0 < \delta < 1$.

REMARK 2.1. *The compatibility assumption* (2.5) *in Theorem 2.1 is important in the analysis. It guarantees that one can take the well-prepared initial datum* (2.4) *instead of the general initial datum, and hence the ansatz is appropriate in this case while, generally speaking, its breakdown will introduce an extra layer of mixing of fast time and fast space scales.*

For the drift-diffusion-Poisson system with much more restrictive assumptions on the doping profile, Gasser, Hsiao, Wang and Markowich [25], Jüngel and Peng [45], and Schmeiser and Wang [71] studied the corresponding small parameters limits problems. For quasi-neutral limit in the case of more general doping profile for drift-diffusion-Poisson system, see [83]. Further applications on electro-hydrodynamics, see [48, 81].

3. Applications of Hsiao's Plasma PDE theory to Euler-Maxwell equations. The most important applications of Hsiao's PDE theory on semiconductor and plasma is to study the Euler-Maxwell (E-M) system on plasma. Usually, it takes the form of the compressible Euler equations forced by the electromagnetic field, which is governed by the Maxwell equation (see [7, 60, 78])

$$\begin{cases} \partial_t n_\nu + \operatorname{div}(n_\nu u_\nu) = 0, \\ m_\nu \partial_t (n_\nu u_\nu) + m_\nu \operatorname{div}(n_\nu u_\nu \otimes u_\nu) + \nabla p_\nu = q_\nu n_\nu (E + \gamma u_\nu \times B) - m_\nu \dfrac{n_\nu u_\nu}{\tau_\nu}, \\ \partial_t \mathcal{E}_\nu + \operatorname{div}(\mathcal{E}_\nu u_\nu + p_\nu u_\nu) = q_\nu n_\nu u_\nu E - \dfrac{n_\nu (\mathcal{I}_\nu - \mathcal{I}_*)}{\tau_\nu} - m_\nu \dfrac{n_\nu |u_\nu|^2}{\tau_\nu}, \\ \gamma \lambda^2 \partial_t E - \nabla \times B = -\gamma (q_i n_i u_i + q_e n_e u_e), \quad \lambda^2 \operatorname{div} E = n_i - n_e, \\ \gamma \partial_t B + \nabla \times E = 0, \quad \nabla \cdot B = 0, \quad \nu = e, i, \quad (t,x) \in \mathbb{R}^+ \times \Omega, \end{cases} \tag{3.1}$$

where $\Omega = \mathbb{R}^3$ or $\mathbb{T}^3$, in which $\mathbb{T}^3 = \left(\dfrac{\mathbb{R}}{2\pi}\right)^3$ stands for a torus in $\mathbb{R}^3$. Here, the unknowns $n_i > 0$, $u_i \in \mathbb{R}^3$ and $\theta_i > 0$ (respectively, $n_e > 0$, $u_e \in \mathbb{R}^3$ and $\theta_e > 0$) are the density, the velocity and the absolute temperature of the ion (respectively, the electron), $E \in \mathbb{R}^3$ is the electric field and $B \in \mathbb{R}^3$ is the magnetic field. Moreover, the functions

$$p_\nu = n_\nu \theta_\nu, \quad \mathcal{E}_\nu = n_\nu \left(\mathcal{I}_\nu + \frac{1}{2}|u_\nu|^2 \right) \quad \text{and} \quad \mathcal{I}_\nu = \frac{3}{2}\theta_\nu, \quad \nu = i, e,$$

denote the pressure, the total energy and the internal energy, respectively. The constants $q_i = 1$, $q_e = -1$, $\theta_* > 0$ and $\mathcal{I}_* = \dfrac{3}{2}\theta_*$ represent the charge of ions, the charge of electrons, the background temperature and the background internal energy, respectively.

Roughly speaking, the works on the E-M system in this section can be regarded as the extensions or applications of Hsiao's PDE theory on semiconductor and plasma. See [4, 6, 10, 25, 28], [33]-[42], [68, 71, 75, 79, 82] and the references therein.

The physical parameters $\tau_\nu > 0$ stands for the momentum relaxation time, $m_\nu > 0$ stands for the mass of charged particle, $\lambda > 0$ stands for the scaled Debye length and $\gamma > 0$ can be chosen to be proportional $\dfrac{1}{c}$, where $c = (\varepsilon_0 \mu_0)^{-\frac{1}{2}}$ is the speed of light, with ε_0 and μ_0 being the vacuum permittivity and permeability.

There are many mathematical investigations in numerical simulations [11], the asymptotic limits with small parameters and the stability of equilibrium solutions for E-M systems.

3.1. The small parameter limits. It is well-known that the physical parameters in system (3.1) are small compared to the physical size of the known variables. Thus, regarding τ_ν, m_ν, γ and λ as singular perturbation parameters, we can study the limits in the system (3.1) as these parameters tend to zero or infty. The limit $\lambda \to 0$ leads to $n_e = n_i$, which is the quasi-neutrality of the plasma. Hence, $\lambda \to 0$ is called the quasi-neutral limit. Also, $\tau_\nu \to 0$, $\gamma \to 0$, $m_e/m_i \to 0$ and $m_i/m_e \to \infty$ are physically called the zero-relaxation limit [32], the non-relativistic limit [63], the zero-mass-electrons limits [27] and the Infinity-Ion-Mass (I-I-M) limit [19], respectively. For the other physical meaning of the dimensionless parameters λ and γ, we refer to [5] for a similar choice of the scaling in the Vlasov-Maxwell equations.

The first mathematical study of the one-dimensional simplified isentropic E-M system is due to Chen-Jerome-Wang [8], where the global existence of entropy solutions is proved by the compensated compactness argument. After that, Peng-Wang [63, 65] prove that the limit $\gamma \to 0$ is the one-fluid compressible Euler-Poisson system and the limit $\lambda \to 0$ is the e-MHD equations. Later on, Peng-Wang [64] prove also that the combined non-relativistic and quasi-neutral limit $\gamma = \lambda^2 \to 0$ is the one-fluid incompressible Euler equations. The justification of these limits is rigorous for smooth periodic solutions in time intervals independent of the parameters γ and λ. Furthermore, the two-fluid E-M equations are studied in [3], where the formal asymptotic analysis is performed to derive a hierarchy of models for plasmas. Next, Peng-Wang [66] study independently, by means of asymptotic expansions, the zero-relaxation limit $\tau_\nu \to 0$, the non-relativistic limit $\gamma \to 0$ and the combined non-relativistic and quasi- neutral limit $\gamma = \lambda^2 \to 0$. Recently, Feng-Li-Wang prove that the I-I-M limit $m_i/m_e \to \infty$ is the one-fluid compressible non-isentropic Euler-Maxwell (N-E-M) equations [19].

We define the small parameter $\varepsilon = 1/\sqrt{m_i}$. Thus the I-I-M limit means letting $\varepsilon \to 0$. By keeping $m_\nu > 0$ and taking the other parameters to be unity, for $n_\nu > 0$ and $\theta_\nu > 0$, system (3.1) is turned into

$$\begin{cases} \partial_t n_\nu + \operatorname{div}(n_\nu u_\nu) = 0, \\ m_\nu \partial_t u_\nu + m_\nu (u_\nu \cdot \nabla) u_\nu + \dfrac{1}{n_\nu} \nabla (n_\nu \theta_\nu) = q_\nu (E + u_\nu \times B) - m_\nu u_\nu, \\ \partial_t \theta_\nu + \dfrac{2}{3} \theta_\nu \operatorname{div} u_\nu + u_\nu \cdot \nabla \theta_\nu = -(\theta_\nu - \theta_*), \\ \partial_t E - \nabla \times B = n_e u_e - n_i u_i, \quad \operatorname{div} E = n_i - n_e, \\ \partial_t B + \nabla \times E = 0, \quad \operatorname{div} B = 0, \quad \nu = i, e, \quad (t, x) \in (0, +\infty) \times \mathbb{T}^3, \end{cases} \tag{3.2}$$

supplemented by the following initial condition

$$(n_\nu, u_\nu, \theta_\nu, E, B)|_{t=0} = (n_{\nu,0}, u_{\nu,0}, \theta_{\nu,0}, E_0, B_0), \quad \nu = i, e, \quad x \in \mathbb{T}^3, \tag{3.3}$$

satisfying the compatibility condition

$$\operatorname{div} E_0 = n_{i,0} - n_{e,0}, \quad \operatorname{div} B_0 = 0, \quad x \in \mathbb{T}^3. \tag{3.4}$$

Obviously, $(n_\nu, u_\nu, \theta_\nu, E, B) = (1, \mathbf{0}, \theta_*, \mathbf{0}, B_*)$ is a constant equilibrium state of system (3.2) in which $B_* \in \mathbb{R}^3$ is any constant vector. The two-fluid N-E-M system

(3.2) is symmetrizable hyperbolic for $n_\nu, \theta_\nu > 0$, then according to the classical result of Kato [46], the periodic problem (3.2)-(3.4) has a unique local smooth solution when the initial data are smooth.

It is well known that in plasma physics, ions move much more slowly than electrons. Thus, based on physical hypothesis

$$n_i = b(x), \quad u_i = 0 \quad \text{and} \quad \theta_i = \theta_*, \tag{3.5}$$

system (3.2) becomes the one-fluid N-E-M system

$$\begin{cases} \partial_t n_e + \operatorname{div}(n_e u_e) = 0, \\ \partial_t u_e + (u_e \cdot \nabla) u_e + \dfrac{1}{n_e}\nabla(n_e\theta_e) = -(E + u_e \times B) - u_e, \\ \partial_t \theta_e + \dfrac{2}{3}\theta_e \operatorname{div} u_e + u_e \cdot \nabla\theta_e = -(\theta_e - \theta_*), \\ \partial_t E - \nabla \times B = n_e u_e, \quad \operatorname{div} E = b(x) - n_e, \\ \partial_t B + \nabla \times E = 0, \quad \operatorname{div} B = 0, \quad (t,x) \in (0,+\infty) \times \mathbb{T}^3. \end{cases} \tag{3.6}$$

Due to the fact that ions are much heavier than electrons, we let the ratio $m_e/m_i \to 0$. There are two different ways to investigate this limiting procedure. One is setting $m_i = 1$ and letting $m_e \to 0$, which is called the zero-electron-mass limit (see [1, 2, 27, 44, 89, 90] and references therein). The other is setting $m_e = 1$ and letting $m_i \to +\infty$, which is called the I-I-M limit, and was recently introduced in [86].

3.1.1. Results on the local-in-time convergence. For the local-in-time convergence, it is not necessary to introduce the velocity and temperature dissipation terms in (3.2). Hence, (3.2)-(3.4) becomes

$$\begin{cases} \partial_t n_i^\varepsilon + \operatorname{div}(n_i^\varepsilon u_i^\varepsilon) = 0, \\ \partial_t u_i^\varepsilon + (u_i^\varepsilon \cdot \nabla) u_i^\varepsilon + \varepsilon^2 \dfrac{1}{n_i^\varepsilon}\nabla(n_i^\varepsilon\theta_i^\varepsilon) = \varepsilon^2(E^\varepsilon + u_i^\varepsilon \times B^\varepsilon), \\ \partial_t \theta_i^\varepsilon + \dfrac{2}{3}\theta_i^\varepsilon \operatorname{div} u_i^\varepsilon + u_i^\varepsilon \cdot \nabla\theta_i^\varepsilon = 0, \\ \partial_t n_e^\varepsilon + \operatorname{div}(n_e^\varepsilon u_e^\varepsilon) = 0, \\ \partial_t u_e^\varepsilon + (u_e^\varepsilon \cdot \nabla) u_e^\varepsilon + \dfrac{1}{n_e^\varepsilon}\nabla(n_e^\varepsilon\theta_e^\varepsilon) = -(E^\varepsilon + u_e^\varepsilon \times B^\varepsilon), \\ \partial_t \theta_e^\varepsilon + \dfrac{2}{3}\theta_e^\varepsilon \operatorname{div} u_e^\varepsilon + u_e^\varepsilon \cdot \nabla\theta_e^\varepsilon = 0, \\ \partial_t E^\varepsilon - \nabla \times B^\varepsilon = n_e^\varepsilon u_e^\varepsilon - n_i^\varepsilon u_i^\varepsilon, \quad \operatorname{div} E^\varepsilon = n_i^\varepsilon - n_e^\varepsilon, \\ \partial_t B^\varepsilon + \nabla \times E^\varepsilon = 0, \quad \operatorname{div} B^\varepsilon = 0, \quad (t,x) \in (0,+\infty) \times \mathbb{T}^3, \end{cases} \tag{3.7}$$

with the initial condition

$$(n_\nu^\varepsilon, u_\nu^\varepsilon, \theta_\nu^\varepsilon, E^\varepsilon, B^\varepsilon)|_{t=0} = (n_{\nu,0}^\varepsilon, u_{\nu,0}^\varepsilon, \theta_{\nu,0}^\varepsilon, E_0^\varepsilon, B_0^\varepsilon), \quad \nu = i, e, \quad x \in \mathbb{T}^3, \tag{3.8}$$

which satisfies the compatibility condition

$$\operatorname{div} E_0^\varepsilon = n_{i,0}^\varepsilon - n_{e,0}^\varepsilon, \quad \operatorname{div} B_0^\varepsilon = 0, \quad x \in \mathbb{T}^3. \tag{3.9}$$

By the classic theory of Kato [46] for the symmetrizable hyperbolic system, we have

PROPOSITION 3.1 (Local existence of smooth solutions, see [46, 58]). *Let $s \geq 3$ be an integer. Suppose $(n^\varepsilon_{\nu,0}, u^\varepsilon_{\nu,0}, \theta^\varepsilon_{\nu,0}) \in H^s(\mathbb{T}^3)$ with $n^\varepsilon_{\nu,0}, \theta^\varepsilon_{\nu,0} \geq 2\kappa$ for some given constant $\kappa > 0$ independent of ε. Then there is $T^\varepsilon_e > 0$ such that problem* (3.7)-(3.9) *admits a unique smooth solution $(n^\varepsilon_\nu, u^\varepsilon_\nu, \theta^\varepsilon_\nu, E^\varepsilon, B^\varepsilon)$ defined on the time interval $[0, T^\varepsilon_e]$, which satisfies $n^\varepsilon_\nu, \theta^\varepsilon_\nu \geq \kappa$ and*

$$(n^\varepsilon_\nu, u^\varepsilon_\nu, \theta^\varepsilon_\nu, E^\varepsilon, B^\varepsilon) \in C([0, T^\varepsilon_e]; H^s(\mathbb{T}^3)) \cap C^1([0, T^\varepsilon_e]; H^{s-1}(\mathbb{T}^3)), \quad \nu = i, e.$$

3.1.2. Asymptotic expansion. We look for an approximation of solution $(n^\varepsilon_\nu, u^\varepsilon_\nu, \theta^\varepsilon_\nu, E^\varepsilon, B^\varepsilon)$ to (3.7) under the form of a power series in ε. Assume that the initial data $(n^\varepsilon_{\nu,0}, u^\varepsilon_{\nu,0}, \theta^\varepsilon_{\nu,0}, E^\varepsilon_0, B^\varepsilon_0)$ admit an asymptotic expansion with respect to ε,

$$(n^\varepsilon_{\nu,0}, u^\varepsilon_{\nu,0}, \theta^\varepsilon_{\nu,0}, E^\varepsilon_0, B^\varepsilon_0)(x) = \sum_{j\geq 0} \varepsilon^{2j} \left(\bar{n}^j_\nu, \bar{u}^j_\nu, \bar{\theta}^j_\nu, \bar{E}^j, \bar{B}^j\right)(x), \quad \nu = i, e, \tag{3.10}$$

where $\left(\bar{n}^j_\nu, \bar{u}^j_\nu, \bar{\theta}^j_\nu, \bar{E}^j, \bar{B}^j\right)_{j\geq 0}$ are sufficiently smooth. Then we make the following ansatz,

$$(n^\varepsilon_\nu, u^\varepsilon_\nu, \theta^\varepsilon_\nu, E^\varepsilon, B^\varepsilon)(t,x) = \sum_{j\geq 0} \varepsilon^{2j} \left(n^j_\nu, u^j_\nu, \theta^j_\nu, E^j, B^j\right)(t,x), \quad \nu = i, e. \tag{3.11}$$

Then plugging (3.11) into system (3.7) and comparing the coefficients of ε, we obtain

PROPOSITION 3.2 (see [19]). *Assume* (3.10), *in which $\left(\bar{n}^j_\nu, \bar{u}^j_\nu, \bar{\theta}^j_\nu, \bar{E}^j, \bar{B}^j\right)_{j\geq 0}$ are smooth enough. And let the conditions in Proposition 3.1 hold. Then there exists a positive time $T_a > 0$, which is independent of ε, such that there exists a unique asymptotic expansion in the form of* (3.11) *with profiles $\left(n^j_\nu, u^j_\nu, \theta^j_\nu, E^j, B^j\right)_{j\geq 0}$ defined on $[0, T_a] \times \mathbb{T}^3$ up to any order of ε.*

3.1.3. Error estimates and main results. Let $m \geq 1$ be a fixed integer and denote the approximate solution of order m by

$$\left(n^m_{\nu,\varepsilon}, u^m_{\nu,\varepsilon}, \theta^m_{\nu,\varepsilon}, E^m_\varepsilon, B^m_\varepsilon\right)(t,x) = \sum_{j=0}^m \varepsilon^{2j} \left(n^j_\nu, u^j_\nu, \theta^j_\nu, E^j, B^j\right)(t,x),$$

where $\left(n^j_\nu, u^j_\nu, \theta^j_\nu, E^j, B^j\right)_{0\leq j\leq m}$ are constructed in the previous subsection. We define the remainders $(R^{\varepsilon,m}_{n_\nu}, R^{\varepsilon,m}_{u_\nu}, R^{\varepsilon,m}_{\theta_\nu}, R^{\varepsilon,m}_E)$ by

$$\begin{cases}
\partial_t n^m_{i,\varepsilon} + \operatorname{div}\left(n^m_{i,\varepsilon} u^m_{i,\varepsilon}\right) = R^{\varepsilon,m}_{n_i}, \\
\partial_t u^m_{i,\varepsilon} + \left(u^m_{i,\varepsilon}\cdot\nabla\right) u^m_{i,\varepsilon} + \varepsilon^2 \left(\dfrac{\theta^m_{i,\varepsilon}}{n^m_{i,\varepsilon}} \nabla n^m_{i,\varepsilon} + \nabla\theta^m_{i,\varepsilon}\right) - \varepsilon^2 \left(E^m_\varepsilon + u^m_{i,\varepsilon} \times B^m_\varepsilon\right) = R^{\varepsilon,m}_{u_i}, \\
\partial_t \theta^m_{i,\varepsilon} + \dfrac{2}{3}\theta^m_{i,\varepsilon} \operatorname{div} u^m_{i,\varepsilon} + u^m_{i,\varepsilon}\cdot\nabla\theta^m_{i,\varepsilon} = R^{\varepsilon,m}_{\theta_i}, \\
\partial_t n^m_{e,\varepsilon} + \operatorname{div}\left(n^m_{e,\varepsilon} u^m_{e,\varepsilon}\right) = R^{\varepsilon,m}_{n_e}, \\
\partial_t u^m_{e,\varepsilon} + \left(u^m_{e,\varepsilon}\cdot\nabla\right) u^m_{e,\varepsilon} + \dfrac{\theta^m_{e,\varepsilon}}{n^m_{e,\varepsilon}} \nabla n^m_{e,\varepsilon} + \nabla\theta^m_{e,\varepsilon} + E^m_\varepsilon + u^m_{e,\varepsilon} \times B^m_\varepsilon = R^{\varepsilon,m}_{u_e}, \\
\partial_t \theta^m_{e,\varepsilon} + \dfrac{2}{3}\theta^m_{e,\varepsilon} \operatorname{div} u^m_{e,\varepsilon} + u^m_{e,\varepsilon}\cdot\nabla\theta^m_{e,\varepsilon} = R^{\varepsilon,m}_{\theta_e}, \\
\partial_t E^m_\varepsilon - \nabla\times B^m_\varepsilon - n^m_{e,\varepsilon} u^m_{e,\varepsilon} + n^m_{i,\varepsilon} u^m_{i,\varepsilon} = R^{\varepsilon,m}_E, \quad \operatorname{div} E^m_\varepsilon - n^m_{i,\varepsilon} + n^m_{e,\varepsilon} = 0, \\
\partial_t B^m_\varepsilon + \nabla\times E^m_\varepsilon = 0, \quad \operatorname{div} B^m_\varepsilon = 0, \quad (t,x) \in (0,+\infty)\times\mathbb{T}^3.
\end{cases}$$

According to the fact that the approximate solution $\left(n_{\nu,\varepsilon}^m, u_{\nu,\varepsilon}^m, \theta_{\nu,\varepsilon}^m, E_\varepsilon^m, B_\varepsilon^m\right)$ is smooth enough, a direct computation implies

$$\sup_{0\le t\le T_a} \left\|\left(R_{n_\nu}^{\varepsilon,m}, R_{u_\nu}^{\varepsilon,m}, R_{\theta_\nu}^{\varepsilon,m}, R_E^{\varepsilon,m}\right)(t)\right\|_{L^2(\mathbb{T}^3)} \le C\varepsilon^{2m+2}, \quad \nu = e, i. \tag{3.12}$$

Let $(n_\nu^\varepsilon, u_\nu^\varepsilon, \theta_\nu^\varepsilon, E^\varepsilon, B^\varepsilon)$ be the exact local smooth solution obtained in Proposition 3.1. When the convergence holds at $t = 0$, establishing the convergence of the asymptotic expansion (3.11) is to prove that

$$(n_\nu^\varepsilon, u_\nu^\varepsilon, \theta_\nu^\varepsilon, E^\varepsilon, B^\varepsilon) - \left(n_{\nu,\varepsilon}^m, u_{\nu,\varepsilon}^m, \theta_{\nu,\varepsilon}^m, E_\varepsilon^m, B_\varepsilon^m\right) \to 0,$$

and obtain its convergence rate as $\varepsilon \to 0$ on a time interval independent of ε. It follows that the local-in-time convergence for the I-I-M mass limit.

THEOREM 3.1 (see [19]). *Let the conditions in Propositions 3.1 and 3.2 hold. Let $s \ge 3$ and $m \ge 1$ be integers. Assume*

$$\begin{aligned}
&\sum_{\nu=i,e} \left\|\left(n_{\nu,0}^\varepsilon - n_{\nu,\varepsilon}^m(0,\cdot), \theta_{\nu,0}^\varepsilon - \theta_{\nu,\varepsilon}^m(0,\cdot)\right)\right\|_{H^s(\mathbb{T}^3)} \\
&\quad + \frac{1}{\varepsilon}\left\|\left(u_{i,0}^\varepsilon - u_{i,\varepsilon}^m(0,\cdot)\right)\right\|_{H^s(\mathbb{T}^3)} \\
&\quad + \left\|\left(u_{e,0}^\varepsilon - u_{e,\varepsilon}^m(0,\cdot), E_0^\varepsilon - E_\varepsilon^m(0,\cdot), B_0^\varepsilon - B_\varepsilon^m(0,\cdot)\right)\right\|_{H^s(\mathbb{T}^3)} \\
&\le C_1\varepsilon^{2m+2},
\end{aligned} \tag{3.13}$$

where C_1 is a positive constant independent of ε, then there is a positive constant C_2, which depends on T_a but is independent of ε, such that as $\varepsilon \to 0$, we have $T_e^\varepsilon \ge T_a$, and the local smooth solution $(n_\nu^\varepsilon, u_\nu^\varepsilon, \theta_\nu^\varepsilon, E^\varepsilon, B^\varepsilon)$ to the periodic problem (3.7) satisfies

$$\sup_{0\le t\le T_a} \|(n_\nu^\varepsilon, u_\nu^\varepsilon, \theta_\nu^\varepsilon, E^\varepsilon, B^\varepsilon)(t) - \left(n_{\nu,\varepsilon}^m, u_{\nu,\varepsilon}^m, \theta_{\nu,\varepsilon}^m, E_\varepsilon^m, B_\varepsilon^m\right)(t)\|_{H^s(\mathbb{T}^3)} \le C_2\varepsilon^{2m+1}.$$

3.2. stability of constant equilibrium solution. For the 3-d isentropic E-M systems, the existence of global smooth small solutions to the Cauchy problem in $\mathbb{R}^3$ is established for $s \ge 3$ and the asymptotic behaviors of solutions when $s \ge 4$ [77]. By using suitable choices of symmetrizers and energy estimates, the global existence and the long time behaviors of smooth solutions to the periodic problem in $\mathbb{T}^3$ and to the initial value problem in $\mathbb{R}^3$ for $s \ge 3$ are established [67, 61]. By high- and low-frequency decomposition methods, uniform (global) classical solutions to the initial value problem in Besov spaces with critical regularity is constructed [87, 88]. For $s \ge 4$, by the tools of Fourier analysis, the decay rates of global smooth solutions in L^q with $2 \le q \le \infty$ when the time goes to infinity are presented [15, 16]. And for $s \ge 6$, the long-time decay rates of global smooth solutions in $H^{s-2k}(\mathbb{R}^3)$ with $0 \le k \le [s/2]$ are also established [76]. For the three-dimensional one-fluid N-E-M systems, the existence of global smooth small solutions to the Cauchy problem in $\mathbb{R}^3$ is established [23]. They consider (3.6) with the initial condition

$$(n_e, u_e, \theta_e, E, B)|_{t=0} = (n_{e0}, u_{e0}, \theta_{e0}, E_0, B_0), \quad \text{in} \quad \mathbb{R}^3, \tag{3.14}$$

satisfying the compatibility condition

$$\nabla\cdot E_0 = 1 - n_{e0}, \quad \nabla\cdot B_0 = 0, \quad \text{in} \quad \mathbb{R}^3, \tag{3.15}$$

and obtain

THEOREM 3.2 (see [23]). *Let $s \geq 4$. There exist $\delta_0 > 0$ and $C_0 > 0$ such that if*

$$\|(n_{e0} - 1, u_{e0}, \theta_{e0} - 1, E_0, B_0)\|_{H^s(\mathbb{R}^3)} \leq \delta_0,$$

then, problem (3.6) *and* (3.14)-(3.15) *admits a unique global solution* $(n_e, u_e, \theta_e, E, B)$ *with*

$$(n_e - 1, u_e, \theta_e - 1, E, B) \in C^1\big(\mathbb{R}^+; H^{s-1}(\mathbb{R}^3)\big) \cap C\big(\mathbb{R}^+; H^s(\mathbb{R}^3)\big)$$

and

$$\begin{aligned} &\sup_{t\geqslant 0} \|(n_e(t) - 1, u_e(t), \theta_e(t) - 1, E(t), B(t))\|_{H^s(\mathbb{R}^3)} \\ &\leq C_0 \|(n_{e0} - 1, u_{e0}, \theta_{e0} - 1, E_0, B_0)\|_{H^s(\mathbb{R}^3)}. \end{aligned}$$

Moreover, there exist $\delta_1 > 0$ and $C_1 > 0$ such that if

$$\|(n_{e0} - 1, u_{e0}, \theta_{e0} - 1, E_0, B_0)\|_{H^{13}(\mathbb{R}^3)} + \|(u_{e0}, E_0, B_0)\|_{L^1(\mathbb{R}^3)} \leq \delta_1,$$

then the solution $(n_e, u_e, \theta_e, E, B)$ satisfies that for any $t \geq 0$,

$$\begin{aligned} &\|(n_e(t) - 1, \theta_e(t) - 1)\|_{L^q(\mathbb{R}^3)} \leq C_1(1 + t)^{-\frac{11}{4}}, \quad \forall\ 2 \leq q \leq +\infty, \\ &\|(u_e(t), E(t))\|_{L^q(\mathbb{R}^3)} \leq C_1(1 + t)^{-2+\frac{3}{2q}}, \quad \forall\ 2 \leq q \leq +\infty, \\ &\|B(t)\|_{L^q(\mathbb{R}^3)} \leq C_1(1 + t)^{-\frac{3}{2}+\frac{3}{2q}}, \quad \forall\ 2 \leq q \leq +\infty. \end{aligned}$$

Furthermore, the global existence and large time decay rates of smooth solutions to the two-fluid N-E-M system (3.1) was obtained by Wang-Feng-Li [80] in $\mathbb{R}^3$. They study the Cauchy problem (3.2)-(3.4), and obtain that the total densities, total temperatures, and magnetic field of two carriers converge to the equilibrium states at the same rate $(1+t)^{-\frac{3}{2}+\frac{3}{2q}}$ in L^q norm. But both the difference of densities and the difference of temperatures of two carriers decay at the rate $(1+t)^{-2-\frac{1}{q}}$, and the velocity and electric field decay at the rate $(1+t)^{-\frac{3}{2}+\frac{1}{2q}}$. This phenomenon on the charge transport shows the essential difference between the N-E-M and the bipolar isentropic E-M system.

Moreover, for the E-M systems without damping, an additional relation was made to establish such a global existence result for the one-fluid E-M system [26]. And if there is no velocity dissipation in the two-fluid system (3.1), Guo-Ionescu-Pausader [30] proved the global existence of smooth solutions to (3.1) in $\mathbb{R}^3$ with the initial assumption $B_0 = \nabla u_{e,0} = -\nabla u_{i,0}$. For more related topics, we refer to [14, 26] and the references therein.

3.3. stability of non-constant equilibrium solution. All these results in the previous section hold when $b(x)$ is a positive constant. When $b(x)$ is a small perturbation of a constant, the Cauchy problem for compressible E-M systems are considered [56, 85], and the time decay rates of smooth solutions are established. We can also see the similar results for viscosity systems [20, 51]. When $b(x)$ is large, such a stability problem is much more complicated than before. Motivated by the Guo-Strauss's work in [31], by employing an induction argument on the order of the derivatives of solutions, the stabilities of non-constant equilibrium solutions for the isentropic E-M systems [62, 21, 55, 54] and N-E-M systems with temperature diffusion terms [22, 50], respectively. Recently, with the help of choosing a new symmetrizer matrix, the stability of the one-fluid N-E-M systems are considered in [52, 53]. Very recently, for smooth initial data near the non-constant steady state, the global existence and large time convergence of smooth solutions to the periodic problem of the two-fluid N-E-M system was obtained by Feng-Li-Wang [18] in $\mathbb{T}^3$. They consider the periodic problem for the two-fluid N-E-M system :

$$\begin{cases} \partial_t n_\nu + \nabla\cdot(n_\nu u_\nu) = 0, \\ \partial_t u_\nu + (u_\nu\cdot\nabla)u_\nu + \dfrac{1}{n_\nu}\nabla p_\nu = q_\nu(E + u_\nu\times B) - u_\nu, \\ \partial_t\theta_\nu + u_\nu\cdot\nabla\theta_\nu + \theta_\nu\nabla\cdot u_\nu = \dfrac{1}{2}|u_\nu|^2 - (\theta_\nu - 1), \\ \partial_t E - \nabla\times B = n_e u_e - n_i u_i, \quad \nabla\cdot E = n_i - n_e + b(x), \\ \partial_t B + \nabla\times E = 0, \quad \nabla\cdot B = 0, \quad \nu = e, i, \quad (t,x)\in\mathbb{R}^+\times\mathbb{T}^3, \end{cases} \tag{3.16}$$

with the initial condition (3.3) which satisfies the compatibility condition

$$\nabla\cdot E_0 = n_{i0} - n_{e0} + b(x), \quad \nabla\cdot B_0 = 0, \quad x\in\mathbb{T}^3, \tag{3.17}$$

and obtain

THEOREM 3.3 (see [18]). *Let $s\geq 3$ and (3.17) hold. Let $\bar{B}\in\mathbb{R}^3$ be any given constant vector and $(\bar{n}_\nu, 0, 1, \bar{E}, \bar{B})$ be an equilibrium solution of (3.16) satisfying $\bar{n}_\nu \geq$ const. > 0. Then there exist constants $\delta_0 > 0$ and $C > 0$, independent of any given time $t > 0$, such that if*

$$\|(n_{\nu 0} - \bar{n}_\nu, u_{\nu 0}, \theta_{\nu 0} - 1, E_0 - \bar{E}, B_0 - \bar{B})\|_{H^s(\mathbb{T}^3)} \leq \delta_0, \quad \nu = e, i,$$

periodic problem (3.16) with (3.3) has a unique global solution $(n_\nu, u_\nu, \theta_\nu, E, B)$ satisfying

$$\begin{aligned} &\sum_{\nu=e,i} |||\,(n_\nu(t,\cdot) - \bar{n}_\nu, u_\nu(t,\cdot), \theta_\nu(t,\cdot) - 1)\,|||_s^2 + |||\,\left(E(t,\cdot) - \bar{E}, B(t,\cdot) - \bar{B}\right)|||_s^2 \\ &\quad + \int_0^t \Bigg(\sum_{\nu=e,i} |||\,(n_\nu(\tau,\cdot) - \bar{n}_\nu, u_\nu(\tau,\cdot), \theta_\nu(\tau,\cdot) - 1)\,|||_s^2 + |||E(\tau,\cdot) - \bar{E}|||_{s-1}^2 \\ &\qquad + |||\partial_\tau B(\tau,\cdot)|||_{s-2}^2 + |||\nabla B(\tau,\cdot)|||_{s-2}^2\Bigg) d\tau \\ &\leq C\sum_{\nu=e,i} ||\left(n_{\nu 0} - \bar{n}^\nu, u_{\nu 0}, \theta_{\nu 0} - 1, E_0 - \bar{E}, B_0 - \bar{B}\right)||_{H^s(\mathbb{T}^3)}^2, \quad \forall\, t\geq 0. \end{aligned} \tag{3.18}$$

Furthermore,

$$\lim_{t\to\infty} |||(n_\nu(t) - \bar{n}_\nu, u_\nu(t), \theta_\nu(t) - 1)|||_{s-1} = 0, \quad \nu = e, i, \tag{3.19}$$

$$\lim_{t\to\infty} |||E(t) - \bar{E}|||_{s-1} = 0, \tag{3.20}$$

and

$$\lim_{t\to+\infty} \left(|||\partial_t B(t)|||_{s-2} + |||\nabla B(t)|||_{s-2}\right) = 0, \tag{3.21}$$

where $|||\cdot|||_m$ *is defined as*

$$|||f|||_m = \sqrt{\sum_{k+|\alpha|\le m} \|\partial_t^k \partial^\alpha f\|^2}, \quad \forall f \in B_{m,T}(\mathbb{T}) = \bigcap_{k=0}^{m} C^k\left([0,T], H^{m-k}(\mathbb{T})\right).$$

Acknowledgments. The authors are grateful to Professor Hsiao for her excellent directions. The authors are supported by the the BNSF (1164010, 1132006), NSFC (11771031, 11531010, 11831003), NSF of Qinghai Province (2017-ZJ-908), NSF of Henan Province (162300410084), the Key Research Fund of Henan Province (16A110019), the general project of scientific research project of the Beijing education committee of China (KM202111232008), the research fund of Beijing Information Science and Technology University (2025029).

REFERENCES

[1] G. Alì, L. Chen, A. Jüngel and Y. J. Peng, *The zero-electron-mass limit in the hydrodynamic model for plasmas*, Nonlinear Anal., 72 (2010), pp. 4415–4427.

[2] G. Alì and L. Chen, *The zero-electron-mass limit in the Euler-Poisson system for both well- and ill-prepared initial data*, Nonlinearity, 24 (2011), pp. 2745–2761.

[3] C. Besse, J. Claudel, P. Degond, et al, *A model hierarchy for ionospheric plasma modeling*, Math. Models Methods Appl. Sci., 14 (2004), pp. 393–415.

[4] Y. Brenier, *Convergence of the Vlasov-Poisson system to the incompressible Euler equations*, Comm. Part. Diff. Eqs., 25 (2000), pp. 737–754.

[5] Y. Brenier, N. J. Mauser and M. Puel, *Incompressible Euler and e-MHD as scaling limits of the Vlasov-Maxwell system*, Comm. Math. Sci., 1 (2003), pp. 437–447.

[6] H. Brézis, F. Golse and R. Sentis, *Analyse asymptotique de léquation de Poisson couplée à la relation de Boltzmann. Quasi-neutralité des plasmas*, C. R. Acad. Sci. Paris, 321 (1995), pp. 953–959.

[7] F. Chen, *Introduction to Plasma Physics and Controlled Fusion*, Vol. 1, Plenum Press, New York, 1984.

[8] G. Q. Chen, J. W. Jerome and D. H. Wang, *Compressible Euler-Maxwell equations*, Trans. Theory Stat. Phys., 29 (2000), pp. 311–331.

[9] G. Q. Chen and D. Wang, *Convergence of shock capturing schemes for the compressible Euler–Poisson equation*, Comm. Math. Phys., 179 (1996), pp. 333–364.

[10] S. Cordier and E. Grenier, *Quasineutral limit of an Euler-Poisson system arising from plasma physics*, Comm. Part. Diff. Eqs., 25 (2000), pp. 1099–1113.

[11] P. Degond, F. Deluzet and D. Savelief, *Numerical approximation of the Euler-Maxwell model in the quasineutral limit*, J.Comput. Phys., 231 (2012), pp. 1917–1946.

[12] P. Degond and P. A. Markowich, *On a one-dimensional steady-state hydrodynamic model for semiconductors*, Appl. Math. Lett., 3 (1990), pp. 25–29.

[13] P. Degond and P. A. Markowich, *A steady-state potential flow model for semiconductors*, Ann. Mat. Pura Appl., IV (1993), pp. 87–98.

[14] Y. Deng, A. D. Ionescu and B. Pausade, *The Euler-Maxwell system for electrons: global solutions in 2D*, Arch. Rational Mech. Anal., 225 (2017), pp. 771–871.

[15] R. J. Duan, *Global smooth flows for the compressible Euler-Maxwell system: the relaxation case*, J. Hyper. Diff. Equations, 8 (2011), pp. 375–413.

[16] R. J. Duan, Q. Q. Liu and C. J. Zhu, *The Cauchy problem on the compressible two-fluids Euler-Maxwell equations*, SIAM J. Math. Anal., 44 (2012), pp. 102–133.

[17] S. Engelberg, H. L. Liu and E. Tadmor, *Critical thresholds in Euler-Poisson equations*, Indiana Univ. Math. J., 50 (2001), pp. 109–157.

[18] Y. H. Feng, X. Li and S. Wang, *Stability of non-constant equilibrium solutions for two-fluid non-isentropic Euler-Maxwell systems arising in plasmas*, J. Math. Phys., 59 (2018), 20 pp.

[19] Y. H. Feng, X. Li and S. Wang, *The global convergence of non-isentropic Euler-Maxwell equations via Infinity-Ion-Mass limit*, preprint 2020.

[20] Y. H. Feng and C. M. Liu, *Stability of steady-state solutions to Navier-Stokes-Poisson systems*, J. Math. Anal. Appl., 462 (2018), pp. 1679–1694.

[21] Y. H. Feng, Y. J. Peng and S. Wang, *Stability of non-constant equilibrium solutions for two-fluid Euler-Maxwell systems*, Nonlinear Anal. Real World, 26 (2015), pp. 372–390.

[22] Y. H. Feng, S. Wang and X. Li, *Stability of non-constant steady-state solutions for non-isentropic Euler-Maxwell system with a temperature damping term*, Math. Methods Appl. Sci., 39 (2016), pp. 2514–2528.

[23] Y. H. Feng, S. Wang and S. Kawashima, *Global existence and asymptotic decay of solutions to the non-isentropic Euler-Maxwell system*, Math. Models Methods Appl. Sci., 24 (2014), pp. 2851–2884.

[24] I. M. Gamba, *Stationary transonic solutions of a one-dimensional hydrodynamic model for semiconductor*, Comm. Partial Differential Equations, 17 (1992), pp. 553–577.

[25] I. Gasser, L. Hsiao, P. A. Markowich and S. Wang, *Quasi-neutral Limit of a Nonlinear Drift Diffusion Model for Semiconductors*, J. Math. Anal. Appl., 268 (2002), pp. 184–199.

[26] P. Germain and N. Masmoudi, *Global existence for the Euler-Maxwell system*, Annales Scientifiques De L école Normale Supérieure, 47 (31), 2011.

[27] T. Goudon, A. Jüngel and Y. J. Peng, *Zero-mass-electrons limits in hydrodynamic models for plasmas*, Appl. Math. Lett., 12 (1999), pp. 75–79.

[28] E. Grenier, *Pseudo-differential energy estimates of singular perturbations*, Comm. Pure Appl. Math., 50 (1997), pp. 821–865.

[29] Y. Guo, *Smooth irrotational fluids in the large to the Euler-Poisson system in* $\mathbb{R}^{3+1}$, Comm. Math. Phys., 195 (1998), pp. 249–265.

[30] Y. Guo, A. D. Ionescu and B. Pausade, *Global solutions of the Euler-Maxwell two-fluid system in 3D*, Ann. of Math., 183 (2016), pp. 377–498.

[31] Y. Guo and W. Strauss, *Stability of semiconductor states with insulating and contact boundary conditions*, Arch. Ration. Mech. Anal., 179 (2005), pp. 1–30.

[32] M. L. Hajjej and Y. J. Peng, *Initial layers and zero-relaxation limits of Euler-Maxwell equations*, J. Differential Equations, 252 (2012), pp. 1441–1465.

[33] L. Hsiao and H. L. Li, *The well-posedness and asymptotic of multi-dimensional quantum hydrodynamics*, Acta Mathematica Sci., 29B(3) (2009), pp. 552–568.

[34] L. Hsiao and H. L. Li, *Compressible Navier-Stokes-Poisson equations*, Acta Mathematica Sci., 30B(6) (2010), pp. 1937–1948.

[35] L. Hsiao, H. L. Li, T. Yang and C. Zou, *Compressible non-isentropic bipolar Navier-Stokes-Poisson system in* $\mathbb{R}^3$, Acta Mathematica Sci., 31B(6) (2011), pp. 2169–2194.

[36] L. Hsiao and T. Luo, *Nonlinear diffusive phenomena of solutions for the system of compressible adiabatic flow through porous media*, J. Differ. Equations, 125 (1996), pp. 329–365.

[37] L. Hsiao, T. Luo and T. Yang, *Global BV solutions of compressible Euler equations with spherical symmetry and damping*, J. Differ. Equations, 146 (1998), pp. 203–225.

[38] L. Hsiao, P.A. Markowich and S. Wang, *Quasineutral limit of a standard drift diffusion model for semiconductors*, Science in China Series A, 45 (2002), pp. 33–41.

[39] L. Hsiao, P. A. Markowich and S. Wang, *The asymptotic behavior of globally smooth solutions of the multidimensional isentropic hydrodynamic model for semiconductors*, J. Differ. Equations, 192 (2003), pp. 111–133.

[40] L. Hsiao and S. Wang, *The asymptotic behavior of global solutions to the hydrodynamic model with spherical symmetry*, Nonlinear Anal. TMA, 52 (2003), pp. 827–850.

[41] L. Hsiao and T. Yang, *Asymptotics of initial boundary value problems for hydrodynamic and drift diffusion models for semiconductors*, J. Differ. Equations, 170 (2001), pp. 472–493.

[42] L. Hsiao and K. J. Zhang, *The global weak solution and relaxation limits of the initial boundary value problem to the bipolar hydrodynamic model for semiconductors*, Math. Models Meth. Appl. Sci., 10 (2000), pp. 1333–1361.

[43] A. Jüngel and Y. J. Peng, *A hierarchy of hydrodynamic models for plasmas, Zero-relaxation-time imits*, Comm. Part. Diff. Equ., 24 (1999), pp. 1007–1033.

[44] A. Jüngel and Y. J. Peng, *A hierarchy of hydrodynamic models for plasmas. Zero-electron-mass limits in the drift-diffusion equations*, Ann. Inst. H. Poincaré Anal. Non Linéaire, 17 (2000), pp. 83–118.

[45] A. Jüngel and Y. J. Peng, *A hierarchy of hydrodynamic models for plasmas: Quasi-neutral limits in the drift-diffusion equations*, Asymptot. Anal., 28 (2001), pp. 49–73.

[46] T. Kato, *The Cauchy problem for quasi-linear symmetric hyperbolic systems*, Arch. Rational Mech. Anal., 58 (1975), pp. 181–205.

[47] S. Klainerman and A. Majda, *Singular limits of quasilinear hyperbolic systems with large parameters and the incompressible limit of compressible fluids*, Comm. Pure Appl. Math., 34 (1981), pp. 481–524.

[48] F. C. Li, *Quasineutral limit of the electro-diffusion model arising in electrohydrodynamics*, J. Differ. Equ., 246 (2009), pp. 3620–3641.

[49] H. L. Li, G. J. Zhang and K. J. Zhang, *Algebraic time decay for the bipolar quantum hydrodynamic model*, Math Models Appl Sci., 18(6) (2008), pp. 859–881.

[50] X. Li, S. Wang and Y. H. Feng, *Stability of non-constant steady-state solutions for bipolar non-isentropic Euler-Maxwell equations with damping terms*, Z. Angew. Math. Phys., 67(5) (2016), pp. 27.

[51] X. Li, S. Wang and Y. H. Feng, *Stability of non-constant equilibrium solutions for bipolar full compressible Navier-Stokes-Maxwell systems*, J. Nonlinear Sci., 28 (2018), pp. 2187–2215.

[52] X. Li, S. Wang and Y. H. Feng, *Stability of nonconstant steady-state solutions for 2-fluid nonisentropic Euler-Poisson equations in semiconductor*, Math Meth Appl Sci., 41 (2018), pp. 3588–3604.

[53] C. M. Liu and Y. J. Peng, *Stability of periodic steady-state solutions to a non-isentropic Euler-Maxwell system*, Z. Angew. Math. Phys., 68 (2017), pp. 17.

[54] C. M. Liu, Z. J. Guo and Y. J. Peng, *Global Stability of large steady-states for an isentropic Euler-Maxwell system in* $\mathbb{R}^3$, Comm. Math. Sci., (7) 17 (2019), pp. 1841–1860.

[55] C. M. Liu and Y. J. Peng, *Convergence of a non-isentropic Euler-Poisson system for all time*, J. Math. Pures Appl., 119 (2018), pp. 255–279.

[56] Q. Q. Liu and C. J. Zhu, *Asymptotic stability of stationary solutions to the compressible Euler-Maxwell equations*, Indiana. Univ. Math. J., 62 (2013), pp. 1203–1235.

[57] T. Luo, R. Natalini and Z. P. Xin, *Large-time behavior of the solutions to a hydrodynamic model for semiconductors*, SIAM J. Appl. Math., 59 (1998), pp. 810–830.

[58] A. Majda, *Compressible Fluid flow and systems of conservation laws in several space variables*, Springer-Verlag, New York, 1984.

[59] P. A. Marcati and R. Natalini, *Weak solutions to hydrodynamic model for semiconductors and relaxation to the drift-diffusion equation*, Arch. Rational Mech. Anal., 129 (1995), pp. 129–145.

[60] P. Markowich, C. A. Ringhofer and C. Schmeiser, *Semiconductor equations*, Springer, 1990.

[61] Y. J. Peng, *Global existence and long-time behavior of smooth solutions of two fluid Euler-Maxwell equations*, Ann. Inst. H. Poincaré Anal. Non Linéaire, 29 (2012), pp. 737–759.

[62] Y. J. Peng, *Stability of non-constant equilibrium solutions for Euler-Maxwell equations*, J. Math. Pures Appl., 103 (2015), pp. 39–67.

[63] Y. J. Peng and S. Wang, *Convergence of compressible Euler-Maxwell equations to compressible Euler-Poisson equations*, Chin. Ann. Math. Ser. B, 28 (2007), pp. 583–602.

[64] Y. J. Peng and S. Wang, *Convergence of compressible Euler-Maxwell equations to incompressible Euler equations*, Comm. Part. Diff. Eqs., 33 (2008), pp. 349–376.

[65] Y. J. Peng and S. Wang, *Rigorous derivation of incompressible e-MHD equations from compressible Euler-Maxwell equations*, SIAM J. Math. Anal., 40 (2008), pp. 540–565.

[66] Y. J. Peng and S. Wang, *Asymptotic expansions in two-fluid compressible Euler-Maxwell equations with small parameters*, Discrete and continuous Dynamical systems, 23 (2009), pp. 415–433.

[67] Y. J. Peng, S. Wang and G. L. Gu, *Relaxation limit and global existence of smooth solutions of compressible Euler-Maxwell equations*, SIAM J. Math. Anal., 43 (2011), pp. 944–970.

[68] Y. J. Peng and Y. G. Wang, *Convergence of compressible Euler-Poisson equations to incompressible type Euler equations*, Asymptotic Anal., 41 (2005), pp. 141–160.

[69] W. V. Roosbroeck, *Theory of the flow of electrons and holes in Germanium and other semiconductors*, Bell Syst. Tech. J., 29 (1950), pp. 560–607.

[70] I. Rubinstein, *Electro-diffusion of Ions*, SIAM, Philadelphia, 1990.

[71] C. Schmeiser and S. Wang, *Quasineutral limit of the drift-diffusion model for semiconductors with general initial data*, Math. Models Methods Appl. Sci., 13 (2003), pp. 463–470.

[72] W. Shockley, *The theory of p-n junctions in semiconductors and p-n junction transistors*, Bell. Syst. Tech. J., 28 (1949), pp. 435–489. (Reprinted in Ann. Mat. Pura Appl., 166 (1987), pp. 65–96.)

[73] J. Simon, *Compact sets in the space* $L^p(0,T;B)$, Ann. Mat. Pura. Appl., 146 (1987), pp. 65–96.

[74] A. Sitenko and V. Malnev, *Plasma Physics Theory*, Chapman and Hall, London, 1995.

[75] M. Slemrod and N. Sternberg, *Quasi-neutral limit for the Euler-Poisson system*, J. Nonlinear Sciences, 11 (2001), pp. 193–209.

[76] Y. Ueda and S. Kawashima, *Decay property of regularity-loss type for the Euler-Maxwell system*, Methods Appl. Anal., 18 (2011), pp. 245–267.

[77] Y. Ueda, S. Wang and S. Kawashima, *Dissipative structure of the regularity type and time asymptotic decay of solutions for the Euler-Maxwell system*, SIAM J. Math. Anal., 44 (2012), pp. 2002–2017.

[78] V. Wasiolek, *Uniform global existence and convergence of Euler-Maxwell systems with small*

parameters, Comm. Pure Appl. Anal., 15 (2016), pp. 2007–2021.

[79] S. Wang, *Quasineutral limit of Euler-Poisson system with and without viscosity*, Comm. Part. Diff. Eqs., 29 (2004), pp. 419–456.

[80] S. Wang, Y. H. Feng and X. Li, *The asymptotic behavior of global smooth solutions of bipolar non-isentropic compressible Euler-Maxwell system for plasma*, SIAM J. Math. Anal., 44 (2012), pp. 3429–3457.

[81] S. Wang, L. M. Jiang and C. D. Liu, *Quasi-neutral limit and the boundary layer problem of Planck-Nernst-Poisson-Navier-Stokes equations for electro-hydrodynamics*, J. Differ. Equ., 267 (2019), pp. 3475–3523.

[82] S. Wang and S. Jiang, *The convergence of the Navier-Stokes-Poisson system to the incompressible Euler equations*, Comm. Part. Diff. Equ., 31 (2006), pp. 571–591.

[83] S. Wang and K. Wang, *The mixed layer problem and quasi-neutral limit of the drift-diffusion model for semiconductors*, SIAM J. Math. Anal., 44 (2012), pp. 699–717.

[84] S. Wang, Z. P. Xin and P. A. Markowich, *Quasi-Neutral limit of the Drift Diffusion models for semiconductors: The case of general sign-changing doping profile*, SIAM J. Math. Anal., 37 (2006), pp. 1854–1889.

[85] W. K. Wang and X. Xu, *Global existence and decay of solution for the nonisentropic Euler-Maxwell system with a nonconstant background density*, Z. Angew. Math. Phys., 67:55 (2016).

[86] S. Xi and L. Zhao, *From bipolar Euler-Poisson system to unipolar Euler-Poisson system in the perspective of mass*, `https://arxiv.org/abs/2002.10867`.

[87] J. Xu, *Global classical solutions to the compressible Euler-Maxwell equations*, SIAM J. Math. Anal., 43 (2011), pp. 2688–2718.

[88] J. Xu, J. Xiong and S. Kawashima, *Global well-posedness in critical Besov spaces for two-fluid Euler-Maxwell equations*, SIAM J. Math. Anal., 45 (2013), pp. 1422–1447.

[89] J. Xu and W. A. Yong, *Zero-electron-mass limit of hydrodynamic models for plasmas*, Proc. Roy. Soc. Edinburgh Sect. A, 141 (2011), pp. 431–447.

[90] J. Xu and T. Zhang, *Zero-electron-mass limit of Euler-Poisson equations*, Discrete Contin. Dyn. Syst., 33 (2013), pp. 4743–4768.

[91] B. Zhang, *Convergence of the Godunov scheme for a simplified one-dimensional hydrodynamic model for semiconductor devices*, Comm. Math. Phys., 157 (1993), pp. 1–22.