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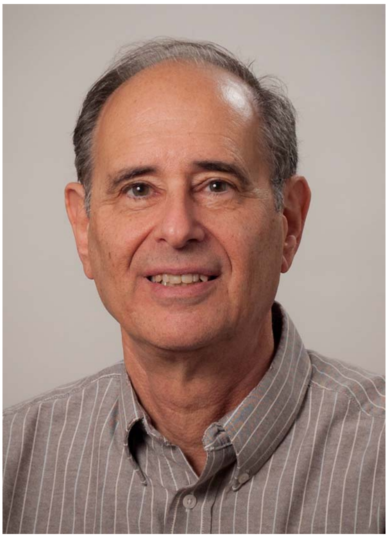
Preface



Bernard Shiffman received his Ph.D. degree in 1968 at U.C. Berkeley under the guidance of Shiing-Shen Chern. His work in complex geometry began at Berkeley with a generalization of a result of Remmert and Stein on removing singularities of complex analytic varieties (1968). This result was later followed up by joint work with Reese Harvey (1974) on characterizing holomorphic k-chains as closed, locally rectifiable currents of type (k, k). In 1971, Shiffman solved a problem (which was also solved independently by Griffiths) posed by Chern at the 1970 ICM by proving an extendability theorem for holomorphic mappings into complex manifolds with nonpositive holomorphic sectional curvature. In a similar vein, Shiffman showed that positive holomorphic line bundles extend holomorphically through analytic subvarieties of complex codimension two (1972).

In joint work with his former student Shanyu Ji and Kollár, Shiffman obtained a global Łojasiewicz inequality for multivariate polynomials over $\mathbb{C}$ or, more generally, over an algebraically closed field with a valuation (1992). And in another joint work with Ji, he showed that Moishezon manifolds can be characterized as compact complex manifolds carrying strictly positive, closed $(1, 1)$-currents with integral de Rham cohomology class (1993), which result is analogous to Kodaira's characterization of projective manifolds.

In his studies of value distribution theory, Shiffman showed in 1975 that the Carlson–Griffiths Second Main Theorem for equidimensional holomorphic mappings holds for meromorphic mappings and for divisors with log canonical singularities, although the term *log canonical* had not come into use at that time. In joint work with Demailly and Lempert in 1994, he showed that holomorphic maps from affine algebraic varieties into algebraic manifolds can be approximated on Runge domains by Nash algebraic maps, and as a consequence the Kobayashi pseudodistance and Kobayashi–Royden infinitesimal metric on algebraic manifolds can be defined using algebraic curves. Shiffman also gave new examples of Kobayashi hyperbolic, projective hypersurfaces in four papers with Zaidenberg (2000–2005).

In joint work with Russakovskii in 1997, Shiffman studied the dynamics of rational self-maps f of complex projective space; they showed for example that under a condition on an intermediate dynamical degree of f, the (normalized) pull-backs by the iterates of f of any probability measure outside an exceptional set converge to a fixed 'equilibrium' measure.

Beginning in 1998, Shiffman started a long-term collaboration with Steve Zelditch in random complex geometry. This occurred at the time that Zelditch (and, independently, Catlin) used the Boutet-de-Monvel–Sjöstrand parametrix for the Bergman (or, Szegő) kernel of a positive Hermitian holomorphic line bundle to calculate asymptotics of the semi-classical Bergman kernel for powers of the line bundle. In the course of that work, Shiffman taught Zelditch the background in complex geometry in a series of tutorials. In the course of these tutorials, they wrote their first joint article, on distribution of zeros of random quantum chaotic sections of positive line bundles, which applied the (then novel) asymptotics to obtain limit laws for currents of integration over zero sets. Pavel Bleher then joined the collaboration for a series of three articles on correlations between zeros. Bleher introduced Shiffman and Zelditch to the Kac–Rice formula in one real dimension, and they generalized the result to all dimensions and to sections of line bundles. Another key result was the first off-diagonal scaling asymptotics for the semi-classical Bergman kernels in terms of the Heisenberg (or, Bargmann–Fock) Bergman kernel. Motivated by Donaldson's paper on embedded symplectic submanifolds, Shiffman–Zelditch then generalized the scaling result in the Kähler setting to all symplectic manifolds. By comparing these results on positive line bundles with classical results of Shepp–Vanderbei on the Kac ensemble, it became clear that there was a very general result in which distribution of zeros tend to equilibrium measures and this was proved in a joint paper in 2003. Other works in this area concern the number of zeros of n independent sections in a domain of an n-manifold and 'holes' in the zero sets (motivated by prior works of Sodin–Tsirelson in complex dimension one).

Shortly after, Shiffman and Zelditch began a collaboration with M. R. Douglas on vacua problems in string theory. Douglas is a prominent string theorist who supplied the physics motivation. Their joint articles used generalized Kac–Rice formulas to study the critical points (relative to smooth connections) of certain holomorphic sections known as super-potentials. In the physics setting, the relevant limit is the high dimension limit with fixed degrees, rather than the prior work on a fixed manifold as the degree of a line bundle tends to infinity.

In related joint work with Bloom, Shiffman showed that the normalized delta-measures of the simultaneous zeros of n i.i.d. random polynomials on $\mathbb{C}^n$, orthonormalized with respect to a measure on a compact set K with some mild regularity conditions, converge almost surely to the equilibrium measure of K as their degrees go to infinity (2007, 2008). Shiffman recently found an asymptotic formula for the variance of the linear statistics of random zeros of sections of holomorphic line bundles of increasing degree (2021), sharpening a result of Sodin in the one variable case and a 2008 joint result with Zelditch (in the hypersurface case). Shiffman also recently collaborated with G. Chirikjian and others on mathematical methods in macromolecular crystallography.