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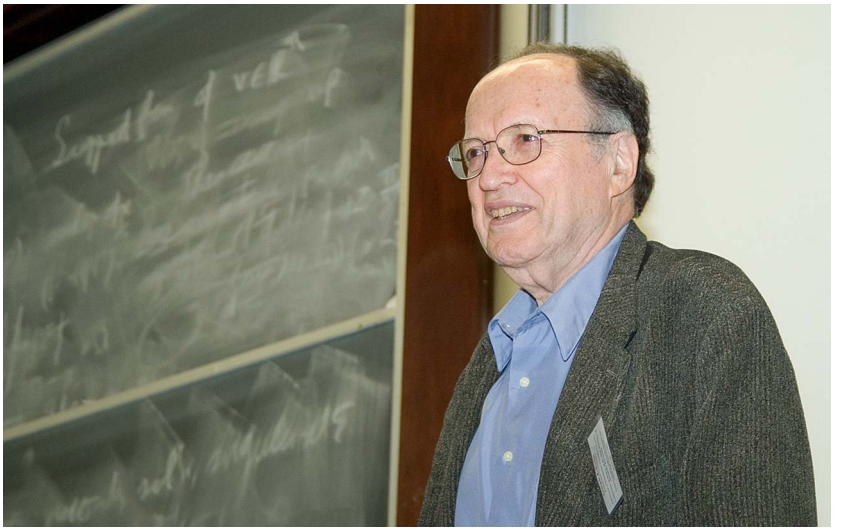
Preface



This issue of Pure and Applied Mathematics Quarterly is dedicated to Professor Joseph J. Kohn, in celebration of his 90th birthday.

Professor Kohn was a founder of the theory of partial differential equations applied to several complex variables. In particular, he pioneered the issue of boundary regularity for the $\bar{\partial}$ equation, and it is to his works that can be traced the origin of fundamental notions such as subelliptic estimates, pseudo-differential operators, microlocal analysis, and multiplier ideal sheaves, which have transformed the mathematical landscape far beyond the theory of several complex variables itself.

It may be appropriate to recall here briefly some contributions of his which have had an exceptional influence, including on the papers in this volume as well as on its editors:

• The boundary regularity of the $\bar{\partial}$-Neumann problem on strongly pseudo-convex domains (1963). This is a really epoch-making work, where subelliptic estimates made their first appearance and were shown to suffice for regularity. A key feature of the underlying equation is that it is a weakly elliptic system with a degenerate direction, which became subsequently a prime motivation for the development of microlocal analysis.

• The $\bar{\partial}_b$-complex (joint work with H, Rossi, 1965). This is a fundamental complex in several complex variables, extending H. Lewy's stunning example

of a linear partial differential equatilon which is not locally solvable. The finiteness of its cohomology led to the introduction of the condition $Z(q)$ much studied since.

• Abstract CR geometry. After the preceding works with Rossi on boundaries of complex manifolds, Professor Kohn was led to develop the microlocal analysis suitable for abstract CR manifolds, which led in turn to embedding results, Hölder estimates, L^p estimates, and the analysis of the operator $\Box_b$.

• The calculus of pseudodifferential operators (joint work with L. Nirenberg, 1965). The loss of fractional derivatives and the proof or regularity in the $\bar{\partial}$-Neumann problem was a strong incentive for Professor Kohn, together with L. Nirenberg, to develop a more systematic treatment, including the handling of commutators. This resulted in this work, which gave the first treatment of pseudo-differential operators and their symbolic calculus. It goes far beyond the theory of singular integral operators of the 1950s, and provides now a basic common tool for seemingly distant theories such as microlocal analysis, semi-classical limits in quantum mechanics, and index theory.

• Multiplier ideal sheaves (1979). This work introduces a strikingly powerful and original way of establishing subelliptic estimates, by constructing ideals of multipliers for which such estimates would hold, and showing how they can be generated successively by differential relations. The method has been applied successfully since in many fields, notably in Kähler geometry and in complex geometry. It led to the study of geometric invariants of boundaries of weakly pseudoconvex domains, and it continues to generate deep and hard questions requiring in particular a blend of real and complex algebraic geometry.

• Hypoellipticity with higher loss of derivatives (2005). This work producing contrary to expectation a system of complex vector fields which is hypoelliptic despite losing more than a full derivative was like a bolt out of the blue sky. A full understanding of this phenomenon remains an important question.

Many of us who had the opportunity also recall with fondness and nostalgia the time that we spent at Princeton under Professor Kohn's guidance and mentorship. There he did not just teach us the statements and proofs of theorems, but a whole approach to mathematics: how to probe in the deepest and most challenging recesses of a question, and rely not on the most flashy theories but rather on the simplest tools. He showed how to do that himself with a method of integration by parts elevated to an art form, supported by a marvelous and unerring intuition which is uniquely his.

The above few remarks should remind everyone in the field how much we all owe to Professor Kohn's works and to his teaching. So it is with particular

appreciation and gratitude that we wish him with this special volume a very happy 90th birthday!

The Editors of the Special PAMQ Issue
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