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Preface

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This Pure and Applied Mathematics Quarterly issue is dedicated to Professor Benedict Gross on his 70th birthday.

Dick Gross was born in South Orange, New Jersey, on June 22, 1950. He attended Pingry High School in New Jersey and received his B.A. from Harvard University in 1971. After spending one year studying music in Indonesia and India, he participated in a master's degree program in mathematics at Oxford University until 1974. He then received his Ph.D. at Harvard University in 1978 under the supervision of John Tate.

Dick Gross started his mathematical career as an assistant professor at Princeton University, an associate professor at Brown University, and eventually returned to Harvard University as a full professor. He has also served as a department chair, Dean of undergraduate education, and Dean of the College. Among many honors and prizes, Gross received a MacArthur Fellowship in 1986, the Cole Prize of the AMS in 1987, and was elected as a member of the National Academy of Sciences in 2004 and of the American Philosophical Society in 2017.

Gross is enthusiastic about teaching mathematics at all levels and communicating great ideas in mathematics to the general public. Gross has formally supervised 37 students, including Henri Darmon, Noam Elkies, Wee Teck Gan, Dipendra Prasad, Douglas Ulmer, and Jiu-Kang Yu. Informally, many younger generations of mathematicians have been greatly influenced by Gross through his mentoring and communications.

Gross has published more than 100 papers. Most of them are in number theory and representation theory. In the early years, 1978–1979, he concentrated on the arithmetic of CM elliptic curves, which includes his thesis on classification and computation of the so-called $\mathbb{Q}$-curves and his new proof of the Chowla–Selberg formula using moduli spaces of Abelian varieties to connect Jacobians of Fermat curves which inspired the notion of absolutely Hodge cycle due to Deligne. In 1980–1981, he developed a theory of p-adic Artin L-functions, which includes his formulation of p-adic Stark conjecture.

In the next period, 1981–1987, he focused on his study of Heegner points on modular curves. His joint work on Gross–Zagier formula has provided one of the essential pieces of evidence for the Birch and Swinnerton-Dyer conjecture. One immediate application of this formula, when combined with an early result of Goldfeld, is a solution to Gauss's class number problem for imaginary quadratic fields. In two papers joint with Kudla (1992) and Schoen (1996) respectively, Gross proposed the first high-dimension extension of the Gross–Zagier formula, which related the heights of Gross–Kudla–Schoen cycles to special values of the first derivative of the triple product L-series.

Around the same period, Gross and his student Prasad developed a theory of branching laws for infinite dimensional representations of orthogonal groups over local and global fields. This theory has been used to unify the Gross–Zagier type formula with the special value formula for L-series in terms of automorphic period integrals. For example, the Gross–Zagier formula is the arithmetic analog of the Waldspurger formula, and the Gross–Kudla–Schoen conjecture is an arithmetic analog of Jacquet's conjecture, proved by Harris–Kudla with refinements given by Gross–Kudla and Ichino. As vast extensions in two papers 2010–2011, joint with Gan and Prasad, Gross proposed branching laws for the classical groups and formulated the so-called GGP conjectures in both automorphic and arithmetic settings for special values and derivatives of Rankin–Selberg type L-series.

In 2009–2011, through separate joint works with Mark Reeder and Edward Frenkel, Gross made the important observation that over the function field $\mathbf{F}_q(t)$, automorphic representations satisfying certain specific local ramification conditions are unique. He then predicted that under the Langlands correspondence for function fields, these unique automorphic representations should give local systems that generalize the Kloosterman local system encoding Kloosterman sums (which were defined by Deligne and studied in depth by Katz). This prediction has led to the notion of rigid automorphic representations, which are the automorphic counterpart of rigid local systems. Gross's insight has opened new ways to construct examples of local systems with dense monodromy in exceptional groups, and has led to applications to the inverse Galois problem.

In two papers in 2013–2015, jointly with Bhargava and Wang, Gross studied arithmetic invariant theory and showed that a positive proportion of hyperelliptic curves of fixed genus g over the rationals ordered by heights have no $\mathbb{Q}$-points.

We want to thank all the people who have contributed to the success of this special volume. Gross's broad interest in mathematics is reflected in the diversity of the contributions that we gathered here. Happy Birthday, Dick.

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