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A lifelong journey through Mathematics

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The Academic Year 1967–68 was quite an unforgettable one, in Rome especially: the student protest which was spreading all over Europe did not spare the University "Sapienza", which at that time was the main state university in the Capital of Italy.

So all the students who enrolled that year found themselves protagonists of a deep intellectual experience more than a set of courses to follow in order to obtain their degree.

Corrado De Concini and I were involved in a peculiar array of meetings, assemblies, student marches and also long hours of mathematics, this last issue possible only if the police was not throwing tear gases against the student protesters.

But we became soon good friends: he arrived at the University late in the morning, when I had already followed two or three lectures and he candidly asked me to give him the notes I took. He studied on them, adding here and there some comments which turned out to be much better that what our professors wrote on the blackboard.

We carried on in this way for three years and a half. The defense of our theses took place in July 1971, mine on the 12th, his on the 15th, both magna cum laude.

He had worked with Silvana Abeasis on spectral sequence in homology of classifying spaces, I preferred algebra, precisely a class of obscure semi-groups, under the influence of I. N. Herstein who visited Rome during the last year of our courses.

Corrado told me that he had been working on a research announcement due to Rothenberg and Steenrod in BAMS (1965).

He admitted after many years that the topic was not so fascinating after all, but since in the announcement there were no proofs, he was attracted by the challenge to produce them. In any case, it was in that period that he decided that he wanted to be a mathematician or at least try.

At that point I lost track of him, as he moved to the University on Warwick in order to obtain a Ph.D. while I remained in Rome with a CNR fellowship to work in group theory.

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When he arrived in Warwick, he was exempted from earning a Master in Science, so he spent all his time studying in order to catch up all the mathematics that we both had not learned in Rome, as the courses were quite superficial.

At Warwick his supervisor happened to be George Lusztig, at that time a young, supertalented and extremely shy young Romanian. When Lusztig first met Corrado, apparently despised his thesis work and gave him a very long list of books to study, among which two had a lasting influence on him, M. Atiyah's *K-Theory* and J. P. Serre's *Représentations linéaires des groups finis*.

Corrado also followed a number of courses. He still keeps the notes of a course by Zeeman on knots, one by Elworthy on differentiable varieties and one by Rourke on cobordism: in this last occasion Corrado gave his first seminar, the topic being the famous Whitney's lemma.

He told me that this talk was a disaster, as he did not speak English fluently enough, he was scared and in the audience he spotted out David Epstein, who he found quite intimidating.

Occasionally he came back to Rome to visit family and friends. He even got married at a certain point and had a daughter, Elena, who was born in Coventry.

During those visits we spent some time talking about our work in mathematics. He was not happy in Warwick as Lusztig was nice with him and very inspiring, but quite distant and Corrado did not feel at ease talking with him. But he kept going to lectures and studying, and finally Lusztig gave him a problem to solve for his thesis, noticing that Corrado had read J. F. Adams's book on Lie Groups and had discovered D. Quillen's work on the Adams's conjecture.

Lusztig gave him the following problem, which had being raised by A. Borel: let G and H be two almost simple connected compact Lie groups which are homotopically equivalent. Are they also isomorphic?

The answer is yes and Corrado proved it using results of Baum and Browder. Unluckily for him, when he was almost ready to discuss his thesis, he found that the paper of Baum and Browder already contained the solution. An involuntary but still evident plagiarism.

This meant that he had to find another problem and write a second thesis, not an easy task.

The second problem came out from the work of Quillen. Quillen among his many deep and spectacular results had calculated the cohomology of $GL(n, F_q)$, F_q being the field with $q = p^r$ elements, with coefficients in $\mathbb{Z}/\ell\mathbb{Z}$, ℓ a prime $\ell \neq p$.

In the paper:

De Concini C. *The mod 2 cohomology of the orthogonal groups over a finite field,* Advances in Math. 27 (1978), no. 3, 191–229,

which contains the results of his thesis, discussed in July 1975, the mod 2 cohomology of the orthogonal groups over finite fields is calculated and this is the only case in which the method of Quillen cannot be applied straightforwardly.

Writing two Ph.D. theses turned out to be very heavy and stressful for Corrado; the job was still not finished, he was exhausted and decided to go back to Italy and pull himself together.

At that point, thanks to C. Procesi, he obtained a position at the SNS in Pisa (1974–75). The salary was very low but there were no teaching duties. Since he needed time to write his second thesis and relax, this solution worked very well.

At Pisa he started a collaboration with S. Buoncristiano whom he had already met at Warwick.

Buoncristiano worked in topology and was interested in a geometric approach to generalized cohomology theories. A series of seminars were organized at the University of Pisa. Corrado lectured on the cobordism groups $\hat{\Omega}_*$ of G-manifolds, $G \subset S_\ell$ a finite permutation group, which were defined in a geometric way. Some explicit computations were performed in the case $G = \mathbb{Z}/p\mathbb{Z}$, p a prime. The results are in the paper:

Buoncristiano S.; De Concini C. *Bordism with codimension-one singularities.* Compositio Math. 35 (1977), no. 1, 65–82.

In the academic year 1975/76 the collaboration with Buoncristiano came to an end.

Corrado left Pisa for Salerno, where he was lecturer of Geometry in the Physics Department of the local University.

After a very troubled year in Pisa, the period in Salerno was instead a very nice one: we started a life together even if I had a position in Rome, as we both broke up with our former sentimental lives and made a new start, after nine years of solid friendship.

In Salerno, Corrado worked with the theoretical physicist G. Vitiello, a very sympathetic, talented and friendly person. This collaboration was fruitful and stimulating, as it produced three joint papers, among which maybe the most interesting is:

De Concini, C.; Vitiello, G. *Spontaneous breakdown of symmetry and group contractions.* Nuclear Phys. B 116 (1976), no. 1, 141–156.

I loved going to Salerno: the town was on the sea and very lively, full of charming restaurants and had a beautiful view on the coast. We would walk along the "lungomare", eat a pizza at the Pizzeria Vicolo della Neve and sometimes make a trip to the Costiera Amalfitana.

At that point a real fantastic thing happened: Procesi moved to Rome and involved Corrado in a new stream of ideas. This has been the starting point of a long lasting collaboration which has been central in Corrado scientific life. Their first joint paper is:

> De Concini, C.; Procesi, C. *A characteristic free approach to invariant theory.* Advances in Math. 21 (1976), no. 3, 330–354.

This was a breakthrough piece of work, which changed the path of mathematics not only for Corrado, but also for myself and other people. It contains the proof of the first and second fundamental theorems of invariant theory for classical groups in a characteristic free approach.

The result in characteristic zero is a classical one and it is attributed in various forms to Cayley, Sylvester and H. Weyl. In positive characteristic the proof is quite different, as there is no linear reductivity. One has to resort to the use of combinatorial methods, partially introduced by Doubillet, Rota and Stein.

Important consequences of this result are versions of the Schur-Weyl duality for $G = GL(V), Sp(V), O(V)$, that is to say that in each case the algebra which commutes with the action of G on $V^{\otimes N}$ is determined in arbitrary characteristic.

Also the combinatorial methods used in this and some of the following papers, was, at least partially, an inspiration to the development of standard monomial theory (see for example:

> Lakshmibai, V.; Seshadri, C. S.; *Geometry of G/P. II. The work of De Concini and Procesi and the basic conjectures.* Proc. Indian Acad. Sci. Sect. A 87 (1978), no. 2, 1–54.)

At the end of 1976, Corrado became an assistant professor at the Department of Mathematics at the University of Pisa. This was a tenure position.

During the following years 1977 and 1978, he devoted himself to finding a basis for the coordinate ring of the variety of m-ples of vectors which span an isotropic subspace of a vector space V of dimension $2m$, with a non degenerate symplectic form.

This allowed him to construct a basis of Young tableau type for the irreducible representations of $Sp(V)$.

I was extremely interested in this method and together we calculated the algebra which commutes with the action of $Sp(V)$ on "traceless tensors", extending in this way to positive characteristic a classical result of H. Weyl.

At the same time Corrado used these ideas to give a basis for the coordinate ring of $Sp(V)$ again in a characteristic free way which implies a characteristic free version of the theorem of Peter Weyl.

These results are contained in the following papers:

1. De Concini, C. *Symplectic standard tableaux.* Adv. in Math. 34 (1979), no. 1, 1–27.
2. De Concini, C.; Strickland, E. *Traceless tensors and the symmetric group.* J. Algebra 61 (1979), no. 1, 112–128.
3. De Concini, C. *Characteristic free decomposition of the coordinate ring of the symplectic group.* Quad. Ricerca Sci., 109, CNR, Rome 1981 "Non commutative structures in algebra and geometric combinatorics," Naples (1978), 121–128.

At this point things were moving really fast and we both accepted an invitation from David Buchsbaum to visit Brandeis University in the spring of 1978 and then again during academic year 1979–80; this time I had a fellowship to visit Harvard University.

Those two years have been extremely interesting and fruitful, thanks to David and other incredible mathematicians like David Kazhdan, David Eisenbud, Victor Kac, Phil Griffiths, David Mumford and many others with which we interacted at various levels.

The atmosphere in the mathematics spread in that area was strongly inspiring and we did our best to take advantage of it.

Various other papers came out among which:

1. De Concini, C.; Eisenbud, D.; Procesi, C. *Young diagrams and determinantal varieties.* Invent. Math. 56 (1980), no. 2, 129–165.

   Quoting the review in Math. Reviews: Let F be a commutative ring and let $m \geq n \geq 1$ be integers. Let $R = F[\{X_{ij}\}_{1 \leq i \leq n, 1 \leq j \leq m}]$ be the polynomial ring on mn variables X_{ij}. Regarding $X = (X_{ij})$ as an $n \times m$ matrix, one can define an action of the group $G = \mathrm{GL}(n, F) \times \mathrm{GL}(m, F)$ on R by the formula $(A, B)X_{ij} = X'_{ij}$, where $A \in \mathrm{GL}(n, F)$, $B \in \mathrm{GL}(m, F)$, and X'_{ij} is the (ij)-th entry of the matrix $A^{-1}XB$. The aim of this paper is to study the arithmetic of the ideals of R that are invariant under the action of G. The method is to exploit the representation theory of G.

2. De Concini, C.; Strickland, E. *On the variety of complexes.* Adv. in Math. 41 (1981), no. 1, 57–77.

   In this paper, using a suitable basis of the coordinate ring of such varieties, we determined the ideal of definition and we proved that for these varieties the irreducible components are normal and Cohen-Macaulay (less precise results were obtained in char 0 by G. Kempf).

3. De Concini, C.; Eisenbud, D.; Procesi. C. *Hodge algebras.* Astérisque, 91. Societé Mathématique de France, Paris, 1982. 87.

In this long article a class of algebras called “Hodge algebras” are introduced, which have a basis of standard monomials for which various properties, like for example being Cohen-Macaulay, can be deduced from combinatorial properties of underline posets using results by Hochster, Stanley and Reisner.

4. De Concini, C.; Lakshmibai, V. *Arithmetic Cohen-Macaulayness and arithmetic normality for Schubert varieties.* Amer. J. Math. 103 (1981), no. 5, 835–850.

   In this paper the bases of standard monomials are used, introduced by Lakshmibai, Musili and Seshadri. A few years later Metha and Ramanathan revolutionized this approach introducing the Frobenius splitting and generalizing enormously this result.

After the years 1979–80, Corrado lost interest in the standard monomials and started to be interested in properties of the Springer fibers and their cohomology especially in type A. The main results in this direction have been the following:

1. De Concini, C.; Kazhdan, D. *Special bases for S_N and $GL(n)$.* Israel J. Math. 40 (1981), no. 3–4, 275–290.

   In this paper one uses the basis for the realization of the irreducible S_N-module associated to a partition Γ in $H^{top}(\mathcal{B}_s, Q)$, the Springer fiber of flags stable under the action of a nilpotent $N \times N$ matrix s of partition Γ and Schur-Weyl duality to obtain a basis of irreducible polynomial representations of $Gl(n)$.

2. De Concini, C.; Procesi, C. *Symmetric functions, conjugacy classes and the flag variety.* Invent. Math. 64 (1981), no. 2, 203–219.

   In this paper one proves an isomorphism of graded rings between the cohomology ring $H^{*/2}(\mathcal{B}_s, Q)$ and the coordinate ring of the non reduced scheme $D \cap \overline{\mathcal{O}_{^t\Gamma}}$, where D denotes the space of diagonal matrices and $\mathcal{O}_{^t\Gamma}$ is the conjugacy class of nilpotent matrices of Jordan partition $^t\Gamma$, the transpose partition of Γ.

In 1980 Corrado was appointed full professor of algebra at the University of Pisa and started to be interested in the problem of compactifying certain homogenous spaces and study their geometry and their applications.

This part of his work resulted in various papers, the first of which is widely known.

The problem is the following. Let G be a semisimple adjoint group, $\sigma : G \to G$ an involution, $H = G^\sigma$. The variety $X = G/H$ is called a symmetric variety. For example if H is a semisimple adjoint group and $G = H \times H$ with $\sigma((h, h')) = (h', h)$, one obtains $X \simeq H$ as a G-homogeneous space with respect to the left and right multiplication by H.

In type A_n this and the example of the orthogonal involutions in which X is the space of nonsingular quadrics in $\mathbb{P}^n$, were studied classically.

The subject is now part of a theory of spherical varieties which was started by Luna-Vust and developed by various people such as Brion, Knop and others.

The first and certainly better known paper which Corrado wrote on the subject is the following:

> De Concini, C.; Procesi, C. *Complete symmetric varieties.* Invariant theory (Montecatini, 1982), 1–44, Lecture Notes in Math., Springer, Berlin, 1983.

In this paper that appeared in the Proceedings of a CIME course, Procesi and Corrado introduced the complete G-equivariant wonderful embedding $\overline{X}$ of X and described its orbit structure. As an application, they gave an algorithm for the computation of certain numbers in enumerative geometry. For example, they computed the famous number 3264 of conics simultaneously tangent to 5 conics in $\mathbb{P}^2$ and also the number 666 841 of quadrics in $\mathbb{P}^3$ simultaneously tangent to 9 quadrics, confirming an old result of Schubert.

After this paper other followed in one way or another related to symmetric varieties. I will just mention a few:

1. De Concini, C.; Procesi, C. *Complete symmetric varieties II. Intersection theory.* Algebraic groups and related topics (Kyoto/Nagoya, 1983), 481–513, Adv. Stud. Pure Math., 6, North-Holland, Amsterdam, 1985.
2. De Concini, C., Procesi, C. *Cohomology of compactifications of algebraic groups.* Duke Math. J. 53 (1986), no. 3, 585–594.

   This paper introduces the so called ring of conditions for a symmetric variety X through the study of equivariant compactifications lying over the wonderful embedding X using certain toric varieties and their fans and a lemma for removing singularities of the intersection of a cycle with a boundary.
3. De Concini, C.; Goresky, M.; MacPherson, R.; Procesi C. *On the geometry of quadrics and their degenerations.* Comment. Math. Helv. 63 (1988), no. 3, 337–413.

   This paper studies some special properties of the variety of complete quadrics.
4. Bifet, E.; De Concini C.; Procesi C. *Cohomology of regular embeddings.* Adv. Math. 82 (1990), no. 1, 1–34.

   This paper explains a method for trying to compute the equivariant cohomology of an embedding and is inspired by the work on symmetric varieties.

I believe that on these matters, it is worth mentioning two more papers in which results in positive characteristic are obtained:

1. Strickland E. *A vanishing theorem for group compactifications.* Math. Ann. 277 (1987), no. 1, 165–171.
2. De Concini, C.; Springer, T. *A compactification of symmetric varieties.* Dedicated to the memory of Claude Chevalley. Transf. Groups 4 (1999), nos. 2–3, 273–300.

In the spring of 1983 Corrado started another intense and very pleasant collaboration. While visiting Paris, Enrico Arbarello, that Corrado had known for a long time, but with whom he had never interacted mathematically, explained to Corrado a conjecture by S. P. Novikov, which gave a solution for the classical Schottky problem, namely a characterization of Jacobians of Riemann surfaces among irreducible principally polarized Abelian varieties in terms of an explicit partial differential equation (the KP equation) satisfied by their Theta functions.

After an intense period of collaboration, using some geometric results of Welters (in turn based on work of Gunning), they managed to give a solution of a weaker form of the problem, giving a characterization of Jacobians in terms of a finite (but large) set of partial differential equations which are in fact the first equations in the so called KP hierarchy.

The work on the Schottky problem kept the two of them involved so much, that they almost forgot to eat or to sleep. I remember walking at sunset from Paris VI, where I was visiting, to Enrico's place in the Marais, calling them desperately under his windows, hoping they had finished for the day, and having no answer, so that I had to go back to my place quite sad. One day they felt so guilty, that they promised me that if they solved the problem, they would invite me at dinner at "La Tour d'Argent" on the Seine. They did solve the problem, but the promise was forgotten! Only after another twenty years Corrado finally remembered that he had to reserve a table there with a fantastic view of Notre Dame.

Soon after this, T. Shiota in the paper:

> Shiota T. *Characterization of Jacobian varieties in terms of soliton equations.* Invent. Math. 83 (1986), no. 2, 333–382.

completely solved the Novikov conjecture and after this Arbarello and Corrado gave a simplified version of the proof.

Very recently Arbarello with Codogni and Pareschi in the paper:

> Arbarello, E.; Codogni, G.; Pareschi, G. *Characterizing Jacobians via the KP equation and via flexes and degenerate trisecants to the Kummer variety; an algebro-geometric approach.* J. Reine Angew. Math. 777 (2021), 251–271,

gave a drastically simplified proof of the result of Shiota and much more.

The main joint works with Arbarello were:

1. Arbarello, E.; De Concini, C. *On a set of equations characterizing Riemann matrices.* Ann. of Math. (2) 120 (1984), no. 1, 119–140.
2. Arbarello, E.; De Concini, C. *Another proof of a conjecture of S.P. Novikov on periods of abelian integrals on Riemann surfaces.* Duke Math. J. 54 (1987), no. 1, 163–178.

Corrado did not work much on these matters anymore, except for the paper:

De Concini, C.; Johnson. R.A. *The algebraic-geometric AKNS potentials.* Ergodic Theory Dynam. Systems. 7 (1987), no. 1, 1–24.

in which potentials for the AKNS system arising from hyperelliptic curves were described.

The eighties have been a very intense period for Corrado and for me as well. First of all we got married.

Corrado gave a talk at the Berkeley ICM in 1986 and I was appointed full professor in algebra at the University of Rome Tor Vergata.

Corrado spent the academic year 1986–87 in Boston visiting Brandeis and Harvard while I stayed in Rome. That period was quite intense mathematically for him. It is worth recalling the papers

1. Arbarello, E.; De Concini C.; Kac, V. G.; Procesi, C. *Moduli spaces of curves and representation theory.* Comm. Math. Phys. 117 (1988), no. 1, 1–36.

   In this paper one tries to study the moduli space of genus g Riemann surfaces (or better an enriched infinite dimensional version of it) as a sort of infinitesimal homogeneous space for the Lie algebra of vector fields on the circle.

2. Arbarello, E.; De Concini, C.; Kac, V. G. *The infinite wedge representation and the reciprocity law for algebraic curves.* Theta functions-Bowdoin 1987, Part 1 (Brunswick, ME, 1987), 171–190, Proc. Sympos. Pure Math., 49, Part 1, Amer. Math. Soc., Providence, RI, 1989.

   In this paper one gives an adelic proof of Weil reciprocity for a complete curve in the spirit of Tate proof of the residue theorem.

In particular during this academic year, Corrado started talking a lot with Victor Kac, not only mathematically, a mutual link which developed in the following years.

In 1987, back in Rome, G. Lusztig visited Rome at my University "Tor Vergata" and Corrado worked together with Procesi and his former advisor on the topology of the so called Springer fibers.

The outcome was the paper:

De Concini, C.; Lusztig, G.; Procesi C. *Homology of the zero-set of a nilpotent vector field on a flag manifold.* J. Amer. Math. Soc. 1 (1988), no. 1, 15–34.

In it, they proved that the Springer fibers have a property (called property S) which tells us that their homology and Chow groups behave as if they can be paved by affine spaces. This result has various applications in representation theory.

Then in 1988 he moved to the Scuola Normale in Pisa and more importantly our son Guglielmo was born. This fact changed a little bit our habits, as the continuous travelling in order to accept invitations to conferences, spend time collaborating with other mathematicians around the world and visiting various institutes had to be adjusted.

So I had to give up for a while this itinerant life, but not completely: Guglielmo travelled with us in many places and started to say some words in Boston when we were visiting the MIT and Harvard, took his first steps at the beautiful park of the IHES in Paris, and starting from when he was three years old, he stayed with a box of colored pencils and a note pad drawing strange animals in the last row of the class rooms in which we gave our talks. He was resigned to his destiny, but he also seemed to have fun, as for example, a talk at the Institute Henry Poincaré came always together with a visit to Disneyworld in Paris.

In the fall of 1989 we both visited Victor Kac at MIT, carrying along with us our son Guglielmo who was one year old and did not like very much travelling by air.

Together with Kac, Corrado worked on the (at that time rather new) theory of quantum groups, with special emphasis on the case in which the quantum parameter q is a root of unity. In their first paper together, they defined a version of a quantum group at a root of unity (different from Lusztig version) which is now called De Concini-Kac quantum enveloping algebra and proved a number of its properties. The paper:

> De Concini, C.; Kac, V. G. *Representations of quantum groups at roots of 1.* Operator algebras, unitary representations, enveloping algebras and invariant theory (Paris, 1989), Progr. Math. 92, 471–506, Birkhauser Boston, Boston, MA, 1990.

Working with Victor Kac has always been quite inspiring, so Corrado and Procesi went on producing interesting results with him and here it is worthwhile to mention the more relevant ones:

1. De Concini, C.; Kac, V. G.; Procesi, C. *Quantum coadjoint action.* J. Amer. Math. Soc. 5 (1992), no. 1, 151–189.
2. De Concini, C.; Kac, V. G.; Procesi, C. *Some quantum analogues of solvable Lie groups.* Geometry and analysis (Bombay 1992), 41–65.
3. De Concini, C.; Procesi C. *Quantum groups.* D-modules, representation theory and quantum groups (Venice, 1992) 31–140, Lecture Notes in Math. 1565, Springer, Berlin, 1993.
4. De Concini, C.; Lyubashenko, V. *Quantum function algebra at roots of 1.* Adv. Math. 108 (1994), no. 2, 205–262.
5. De Concini, C.; Procesi, C. *Quantum Schubert cells and representations at roots of 1.* Algebraic groups and Lie groups, 127–160, Austral. Math. Soc. Lect. Ser., 9, Cambridge Univ. Press, Cambridge, 1997.

This was also the subject of the plenary talk that Corrado gave at the First European Congress of Mathematics, which was held in Paris in 1992 and to which I could not participate being busy in Rome.

At the Scuola Normale in Pisa, Corrado had a number of students both undergraduate and graduate and collaborated with Fabio Fagnani for a while,

More importantly, after a couple of years he had regular mathematical meetings with M. Salvetti, who is a person that he feels very close, concerning the cohomology of generalized braid (and sometimes Coxeter) groups.

His first paper on the subject:

> De Concini, C.; Salvetti, M. *Cohomology of Artin groups; Addendum: The homology type of Artin groups [Math. Res. Lett. 1 (1994), no. 5, 565–577; MR1295551] by Salvetti.* Math. Res. Lett. 3 (1996), no. 2, 293–297.

consisted essentially in the construction of an explicit free resolution of the generalized braid group G associated to a finite Coxeter group W which in turn came out from the combinatorics of the so called Salvetti complex for a complexified hyperplane arrangement (recall that by a theorem of Brieskorn, the complement of the root arrangement is a $K(G,1)$).

This allowed them to make explicit computations of the cohomology of G with coefficients in various local systems. In particular it was possible to show that the Schwarz genus of the Coxeter fibration equals the rank of $W+1$ except for the symmetric group S_n when n is not a prime power (in the case of the symmetric groups, this had been performed already by Vassiliev and had as a consequence the fact that the topological complexity of the problem of solving a degree n polynomial equation is n).

When n is not a prime power, the question turned out to be rather tricky (due to the fact that the greatest common divisors of the binomial coefficient $\binom{n}{i}$, $0 < i < n$ equals 1).

In the case of S_n, $n = 6$, it was finally proved that the Schwarz genus is 5 and this is explained in the paper:

> De Concini, C.; Procesi, C.; Salvetti, M. *On the equation of degree 6.* Comment. Math. Helv. 79 (2004), no. 3, 605–617.

For a general n, the question appears to be not completely understood.

The study of the topology of the complements of hyperplane arrangements which started with the collaboration with Salvetti and the wish to understand Kontsevich knot invariants, soon took Procesi and Corrado to investigate the problem of finding a well behaved (again wonderful!) compactification of the complement of a finite union of subspaces in a projective space $\mathbb{P}(V)$, V a finite dimensional complex vector space.

They defined a series of models, each associated to what is called a building set, in terms of nested sets. Among other things, they used an idea of Fulton and MacPherson to show that the rational homotopy type of a subspace complement is determined by an explicit set of combinatorial data.

In turn, in the case of a root arrangement, this led to a method for the "computation" of the monodromy of a local system on the complement. In 1996 Corrado conjectured that for what is now called the Casimir connection, this leads to a representation of the group G conjugate to the one defined by Lusztig. This conjecture was later claimed by Toledano Laredo, but it is not completely clear if the proof ever appeared.

In any case the results on subspace arrangements appeared in:

1. De Concini, C.; Procesi, C. *Wonderful models of subspace arrangements.* Selecta Math. (N.S.) 1 (1995), no. 3, 459–494.
2. De Concini, C.; Procesi, C. *Hyperplane arrangements and holonomy equations.* Selecta Math. (N.S.) 1 (1995), no. 3, 495–535.

Staying at the Scuola Normale in Pisa became quite tiring for Corrado, not for the place itself as he found the working conditions, his colleagues with whom he could connect both mathematically and socially, both at the Scuola and at the University, and especially the students very stimulating and not for Pisa, as he had there a nice apartment in Via S. Martino which was quite charming.

The real reason was the weekly commuting, as our son was now in school in Rome and I was in Tor Vergata, which implies itself some commuting, being located in the southern outskirts of Rome. Moreover I was really deeply involved in organizing seminars and workshops and running a project in computer algebra, therefore my trips to Pisa were rare.

So, when in 1996 he was offered the chair of algebra at "Sapienza" in Rome, he accepted and that was his final destination until retirement.

In the year 1997 he gave a series of lectures on the then recent work by Givental on mirror symmetry. The outcome was the small booklet:

Bini, G.; De Concini, C.; Polito, M.; Procesi, C. *On the work of Givental relative to mirror symmetry.* Notes of Courses given by teachers at the Scuola Normale Superiore, Pisa 1998, pp. ii+92.

Around that period under the influence of Michèle Vergne who was often coming to Rome, together with Procesi, Corrado got interested in various applications of the theory of hyperplane arrangements and more generally toric arrangements, a notion which had been introduced before by E. J. N. Looijenga.

In particular formulas for Jeffrey-Kirwan residues and for the number of integer points in a rational polytope obtained by M. Brion and by M. Vergne and by A. Szenes and M. Vergne.

Most of the results obtained in this direction including those on Toric arrangements, were collected in the monograph:

> De Concini, C.; Procesi C. *Topics in hyperplane arrangements, polytopes and box-splines.* Universitext. Springer, New York, 2011, xx+384.

In due time the conversations with Vergne developed into an intense and fruitful collaboration aimed mostly at the study of the index of T-equivariant transversally elliptic operators with T a commutative compact Lie group, which form the subject of the papers:

1. De Concini, C.; Procesi, C.; Vergne, M. *Vector partition functions and generalized Dahmen and Michelli spaces.* Transf. Groups 15 (2010), no. 4, 751–773.
2. De Concini, C.; Procesi, C.; Vergne, M. *Vector partition functions and index of transversally elliptic operators.* Transf. Groups 15 (2010), no. 4, 775–811.
3. De Concini, C.; Procesi, C.; Vergne, M. *Infinitesimal index. Cohomology computations.* Transf. Groups 16 (2011), no. 3, 717–735.
4. De Concini, C.; Procesi, C.; Vergne, M. *The infinitesimal index.* J. Inst. Math. Jussieu 12 (2013), no. 2, 297–334.
5. De Concini C.; Procesi, C.; Vergne, M. *Box splines and the equivariant index theorem.* J. Inst. Math. Jussieu 12 (2013), no. 3, 503–544.

These articles contain various results, the most notable is the solution of an old problem posed in:

> M. F. Atiyah, *Elliptic operators and compact groups.* Lecture Notes in Mathematics, vol. 401, Springer, Berlin, 1974.

regarding the range of the index of such a transversally elliptic operators which Atiyah had solved for $T = S^1$.

In the period 2003–2007 Corrado served as President of the Istituto Nazionale di Alta Matematica (INdAM) in Rome, a job for which he did not feel particularly suited.

As a matter of fact when he decided in 2007 not to run for re-election, I decided instead to run myself. After a really tough campaign, I succeeded in becoming the first woman vice president of INdAM, so that for eight years I fought to preserve and improve what Corrado had done, which in my opinion was remarkable.

About toric arrangements he recently collaborated with G. Gaiffi on the problem of constructing "wonderful" compactifications of the complement of a toric arrangement. These compactifications were indeed constructed in the paper:

> De Concini, C.; Gaiffi, G. *Projective wonderful models for toric arrangements.* Adv. Math. 327 (2018), 390–409.

Here, in order to get the results one has to use a mixture of the methods used in the case of subspace arrangements in projective space and of the theory of toric varieties.

These results were later used to describe the rational homotopy type of the complement of a toric arrangement in the sense of Sullivan, by constructing an explicit differential graded algebra in:

> De Concini C.; Gaiffi, G. *A differential algebra and the homotopy type of a complement of a toric arrangement.* Atti Accad. Naz. Lincei Rend. Lincei Mat. Appl. 32 (2021), no. 1, 1–21.

These are some of the main themes treated by Corrado in his by now long activity. To finish I believe that it is worth mentioning few relatively recent papers which for a number of reasons, I find particularly notable in his more recent production:

1. De Concini, C.; Procesi, C.; Reshetikhin, N.; Rosso, M. *Hopf Algebras with trace and representations.* Invent. Math. 161 (2005), no. 1, 1–44.
2. Chirivì, R.; De Concini, C.; Maffei, A. *On normality of cones over symmetric varieties.* Tohoku Math. J. (2) 58 (2006), no. 4, 599–616.
3. De Concini C.; Papi, P.; Procesi, C. *The adjoint representation inside the exterior algebra of a simple Lie Algebra.* Adv. Math. 280 (2015) 21–46.

In 2019 Corrado retired from the University of Rome Sapienza, one year after my own retirement.

Shortly after Corrado was made Professor Emeritus, and he still has scientific and research activities and collaborators.

Moreover in 2021 he was elected President of the "Accademia Nazionale delle Scienze detta dei XL". This role seems quite challenging, as many meetings and conferences have to be organized, plus a lot of fund raising, which is not an easy task. But for someone like him it is more or less impossible to stand idly, especially during a period in which an unexpected pandemic has really strongly tested everybody. In any case a brilliant mind should be used as long as possible, it is a wise thing to do.

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