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Foreword

Claudio Procesi graduated in 1963 from the University of Rome, and obtained his Ph.D. in Mathematics from the University of Chicago, under the guidance of Nathan Herstein. Back to Italy, he soon became full professor in Algebra in the Universities of Lecce and Pisa. In 1975 he moved to Roma, where he held the chair of Algebra until he became Emeritus Professor in 2013.

At the beginning of his activity, Procesi was interested in non-commutative algebra, with particular attention to the study of algebras with polynomial identities and, more specifically, matrix algebras. He introduced the algebra of generic matrices, giving an explicit realization of it. He was then led to the study of the theory of invariants. In a famous work, he determined the ring of invariant functions on the space of $n \times n$ matrices with respect to the simultaneous conjugation action. As an application, he proved the celebrated Procesi-Razmyslov Theorem, which states that all identities of $n \times n$ matrices can be deduced from the Cayley-Hamilton identity.

In the context of the study of invariants, Procesi and De Concini proved the first and second fundamental theorems of the theory of invariants for classical groups in any characteristic. As a byproduct of their approach, one can show that the actions of $GL(V)$ and of the symmetric group S_n on $V^{\otimes n}$ satisfy the double centralizing property. In the case of zero characteristic, this property was obtained by I. Schur in his celebrated thesis (1900). Ideas of this work are at the origin of results on the classification of ideals in the ring of polynomial functions on $n \times m$ matrices, invariant under the action of $GL(n) \times GL(m)$, and the notion of Hodge algebra. The latter notion allows us to reduce the proof of geometric properties of special manifolds (determinants, Schubert and related manifolds) to the study of associated combinatorial objects. These works gave birth to the theory of standard monomials developed by Seshadri, Musili, Lakshmibai and others, and completed by Littelmann.

A few years later, Procesi, in collaboration with H. Kraft, proved the normality of the closure of conjugate classes of matrices. In collaboration with De Concini, Procesi introduced a remarkable equivariant compactification of the homogeneous space G/H, where G is an adjoint semisimple group and H is the subgroup fixed by an involution. This work, initially inspired by classical enumerative results, has had many consequences and applications over the years. In particular, the idea of wonderful compactification plays a central role in problems of classification of spherical manifolds. Furthermore,

its use was central to Lafforgue's proof of the Langlands correspondence for $GL(n)$ in the case of fields of functions.

In the late 1990's, Procesi and De Concini introduced a compactification of the complement of a finite number of linear subspaces in a projective space. As an application, an alternative construction of the link invariant introduced by Kontsevich was given. Such ideas have then been used by Procesi and others in the study of partition functions and in determining the values of the index of a transversally elliptic equivariant operator, solving an old problem dating back to Atiyah. In addition, Procesi made many other important contributions: a proof for $GL(n)$ of the famous Mumford conjecture on reductive groups; the nonlinearity of the group of automorphisms of a free group; some contributions to the study of the cohomology of Artin groups; the study of important topological properties of Springer fibers; the study of the structure and representations of quantum groups at the roots of unity.

Recently, Procesi has been interested in the study of normal forms for non-linear Schrodinger operators and has returned to PI-algebras, publishing an impressive monograph with Aljadeff, Giambruno, and Regev.

Procesi held many Visiting Professor positions (the Lagrange-Michelet chair in 2004, the Powell chair at the University of California San Diego in 2009 to name just two). Procesi was an Invited Speaker at the ICM Helsinki (1978), Algebraic Geometry section. He received several honors, including the Gold medal of *Accademia Italiana delle Scienze detta dei XL*, Mathematics section, and the Feltrinelli Prize of *Accademia dei Lincei*. He has been a member of the *Accademia dei Lincei* since 1987.

Procesi had leading roles in the international mathematical community, and, from 2007 to 2010, he served as Vice-President of the International Mathematical Union. Procesi has been member of important committees, like the Algebra panel for the ICM meetings in Berkeley (1986) and Zurich (1994), the Scientific Committee for the first European Congress of Mathematics (1990), and the Abel Prize Committee 2006/2007.

Procesi was actively committed to the dissemination of Mathematics in the Developing Countries: he organized summer schools from 2003 to 2007 in Ghana, Kenya, Uganda, Tanzania.

Procesi's influence on the Italian mathematical environment cannot be overestimated: he basically introduced Lie theory in Italy. His scientific achievement shows from the very first contributions how fruitful it can be to combine techniques from Algebra, Algebraic Geometry and Combinatorics. This attitude, which is at the basis of what we nowadays call Geometric Representation Theory, was unprecedented in Italy in the '70s.

We are grateful to Claudio for what we have learned from him as students, colleagues and collaborators.

We would like to thank the contributing authors, who have readily accepted our invitation. We believe that huge variety of topics covered in this volume faithfully reflects the scope and profoundness of Claudio's achievement.

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