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Study of impact of COVID-19 on industrial production indices using singular spectrum analysis

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This paper investigates the impact of the COVID-19 pandemic on 8 different indices of industrial production (IIPs) for three major European countries: France, Germany, and the UK. The analysis is based on applying a combination of Singular Spectrum Analysis (SSA) algorithms, in a way that allows for the proper separation of the trend and seasonal subcycles of the IIPs. The main purpose is to illustrate how to carry out the procedure of the correct decomposition by SSA for the specific series. The accurately extracted trends are analysed and the influence of the pandemic is calculated. The results confirm that necessary goods, such as food and utilities, have low income elasticity of demand since the effect of COVID-19 is negligible for these IIPs. However, for the IIPs of less essential products, the negative impact is much more extreme, although the severity varies depending on several factors, which also aligns with the economic theory.

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1. INTRODUCTION

The 2007 credit crunch and the subsequent crisis has been the main economic recession experienced in recent history. However, the COVID-19 pandemic has had a rivalling negative global impact, which has resulted in a similar economic downfall. The permanent income hypothesis [9] may attempt to explain the reactions of consumers to such occurrences by considering the nature of the shock and the public's expectations. However, evaluating the effect on the economy as a whole may prove to be more complicated. Hence, studying the time series of IIPs, in particular, can show changes in various economic sectors and give insight into the responses of economic agents.

The effect of COVID-19 is not globally uniform due to government policies or other factors relating to certain countries. There are also differences across sectors of production, as the public demand for non-essential, or luxury, products is much more sensitive to changes in consumers' disposable incomes, this is described by the economic theory of income elasticity of demand. Therefore, to capture the full extent of the influence of COVID-19 on production, we consider a range of industrial production components across the UK, France, and Germany.

The movement of IIPs is often studied by economists to acquire an understanding of the state of the industry sector, the extent to which a country relies on imports, and the gross domestic product (GDP) as a whole. This information is utilised by governments, policymakers, and central banks, in particular for predicting the development of related economic sectors. Hence, the tendencies and forecasting of IIPs have been studied from many different angles. Research has been undertaken into the adequacy of IIP forecasts via autoregressive models [23], neural network models [19], among many other linear and non-linear models. However, the application of SSA forecasting has proven to be an increasingly popular approach with remarkable results, since the work by [17], in which SSA significantly outperformed ARIMA. Further comparison studies into other forecasting algorithms typically include SSA as a benchmark, see e.g. [18, 22].

The study of catastrophic events has also developed into an area within the field of event studies in economics. Recent papers, such as [1, 2, 4] have focused on how COVID-19 and government reactionary policy changes have affected a range of different economic measures. The relation between COVID-19 and industrial production has been addressed in [7], however, it provides little inference from results, as well as only considering the IIPs in aggregate.

During such events, the economy suffers from shocks, and thus economic time series experience structural breaks. This brings complexity to the form of series and highlights the necessity for the correct application of the SSA techniques for trend extraction and subcycle separation.

In this paper, we study the impact of COVID-19 by decomposing the monthly time series of IIPs into the sum of a trend of complex shape and seasonal subcycles using DerivSSA and Basic SSA, two algorithms from the SSA methodology [11]. A combination of these algorithms allows for accurate trend extraction, hence the severity of the impact of the pandemic can be properly evaluated.

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2. SSA METHODOLOGY

SSA is a methodology comprised of a family of non-parametric methods rooted in the same idea of decomposing a time series into identifiable components. Its flexibility stems from the modifications made to the Basic SSA algorithm [11, 12, 14]. Historically, two separate, yet simultaneously progressing, streams of research have culminated in the present-day methodology of SSA. One set of developments occurred in the Soviet Union and was disclosed after its fall in [6] (unsurprisingly this reference along with other literature from the time are in Russian, however similar English alternatives, such as [5] exist), whilst the other took place in the West and was well publicised, see e.g. [3, 8, 25, 26].

Further, the core version of SSA, Basic SSA, is introduced, along with its modification DerivSSA. The latter is prominently used for decomposing a time series into the sum of components (usually subcycles) that are better separated from each other.

2.1 Basic SSA

A brief overview of the Basic SSA algorithm is provided below, however for an in-depth description of Basic SSA with full technical details see [12, 15, Chapter 1].

Let $F = (x_1, \ldots, x_N)$ be a time series of length N. Given a window length L $(1<L<N)$, a sequence of L-lagged vectors $X_i=(x_i, \ldots, x_{i+L-1})^T$, $i = 1, 2, \ldots, K=N–L+1$, is constructed and composed into the matrix $\mathbf{X} = (x_{i+j-1})_{i,j=1}^{L,K} = [X_1 : \ldots : X_K]$. This matrix has size $L \times K$ and is often called the 'trajectory matrix'. It is a Hankel matrix, which means that all the elements along the antidiagonals $i+j =$ const are equal. The columns X_j of $\mathbf{X}$ can be considered as vectors in the L-dimensional space $\mathbb{R}^L$.

The singular value decomposition (SVD) of $\mathbf{X}$ yields a sum of L matrices of rank 1

$$\mathbf{X} = \sum_{j=1}^{L} \sqrt{\lambda_j} U_j V_j^T ,$$

where λ_j, $j = 1, \ldots, L$, are eigenvalues of the matrix $\mathbf{X}\mathbf{X}^T$ (typically given in descending order), U_j are the corresponding eigenvectors (left-singular vectors), and $V_j = \mathbf{X}^T U_j \sqrt{\lambda_j}$ are factor vectors (right-singular vectors), together they form eigentriples. The SVD is closely related to the eigenvalue decomposition of $\mathbf{X}\mathbf{X}^T$, namely $\mathbf{X}\mathbf{X}^T = \sum_{j=1}^{L} \lambda_j U_j U_j^T$. In the reconstruction stage of Basic SSA, an averaging over the antidiagonals of the matrix, whose columns are projections of X_i on U_j, is performed, which yields some Hankel matrix that is in a one-to-one correspondence with the time series F_j (the j-th elementary component), then $F = \sum_{j=1}^{L} F_j$ is the Basic SSA decomposition of the original time series F into the sum of, usually simple structured, components that can be grouped so that they have some interpretation. For example, we may create groups that correspond to a trend, subcycles of certain frequency, and components with non-regular behaviour. The possibility to swap operations of diagonal averaging and grouping is described in more detail in [11, 12, 15].

2.2 DerivSSA

DerivSSA is a modification of Basic SSA which differs in the way that the trajectory matrix is constructed, see [13] for thorough explanations of the idea and mechanism behind this method. In short, the difference lies in that the trajectory matrix is constructed as $\mathbf{X}_D = [X_1 : \ldots : X_K : X_2 - X_1 : \ldots : X_K - X_{K-1}]$ instead of $\mathbf{X}$, hence changing the SVD, since the eigentriples considered now correspond to the new matrix $\mathbf{X}_D$. Following the rest of the steps in a similar fashion to Basic SSA, we obtain $F = \sum_{j=1}^{L} \tilde{F}_j$, the DerivSSA decomposition of the original time series F, where the elementary components $\tilde{F}_j$ are usually sorted by decreasing frequency. This is due to the fact that the trajectory matrix with the sequential differences causes changes in the component contributions resulting in an increase in the impact of higher frequency components [11, 13].

3. THE DATA

The industrial production data considered in this paper has been acquired from Eurostat, the European Community's official statistical agency. It consists of 24 unadjusted monthly series, covering 3 countries: France, Germany and the UK; over 8 main sectors of industry: Food Products, Chemicals, Basic Metals, Fabricated Metals, Machinery and Equipment, Electrical Equipment, Vehicles (motor vehicles, trailers and semi-trailers), and Utilities (electricity, gas, steam and air conditioning supply). These are shown in Figure 1, where green corresponds to France, blue to Germany, and red to the UK IIPs. The data spans different time periods for the different countries, depending on the availability from the source. The sample period begins in January 1990 for France, in January 1991 for Germany, and in January 1998 for the UK. It ends in October 2020 for the UK, but in November 2020 for both France and Germany.

There have been several papers studying the same 24 IIPs, see e.g. [17, 19, 21, 24], due to their good representation of the industry, since these sectors cover over 50% of industrial production in their respective countries.

4. APPLICATION OF SSA TO IIPS

The choice of algorithm from the SSA methodology is crucial for a correct decomposition of the given time series into the sum of a trend, a seasonal component, and a non-regular component. A good decomposition of the seasonal component into a sum of cycles of different frequencies may also be of interest. Further in this section, we demonstrate how to choose the appropriate algorithm for decomposing the IIPs.

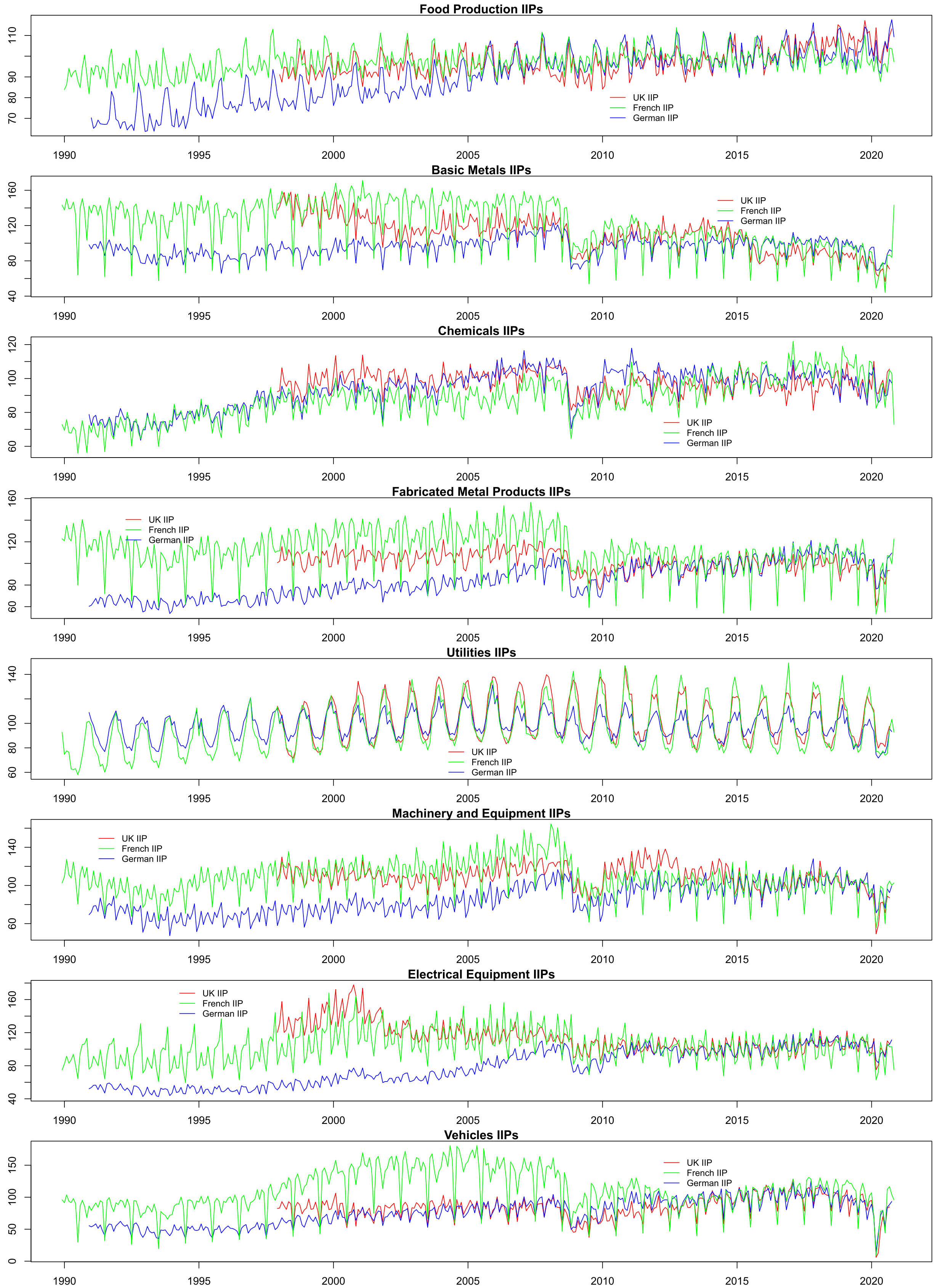

Figure 1. Original time series of IIPs for Germany, France and the UK.

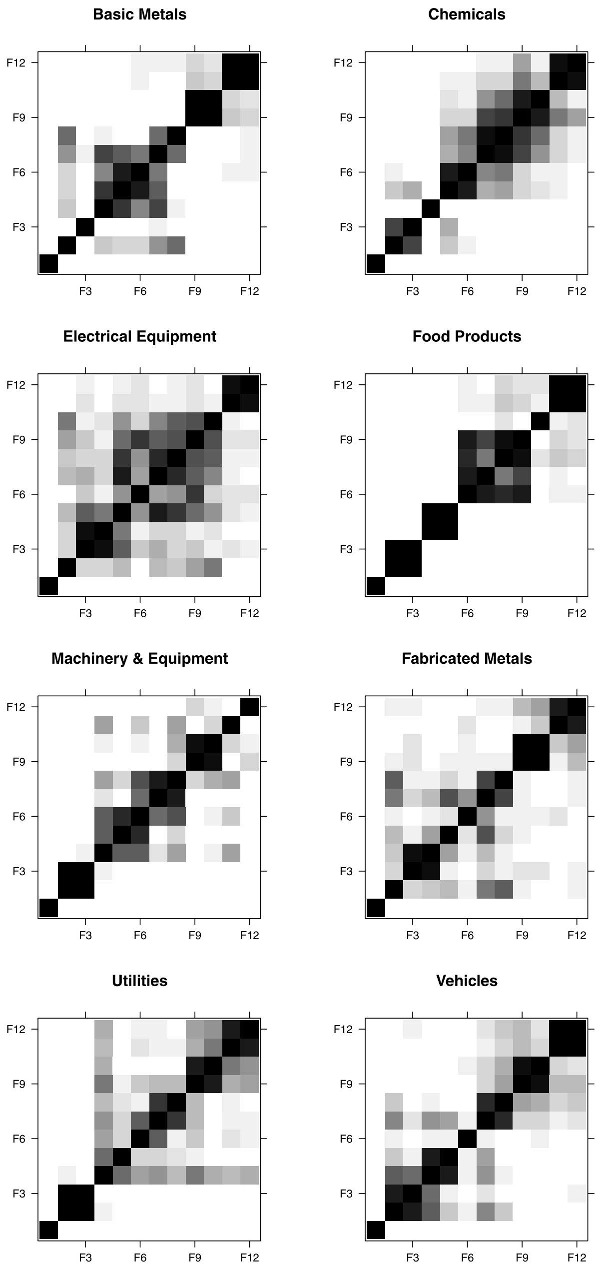

Figure 2. W-correlation matrices of elementary components of the Basic SSA decomposition for eight German IIPs.

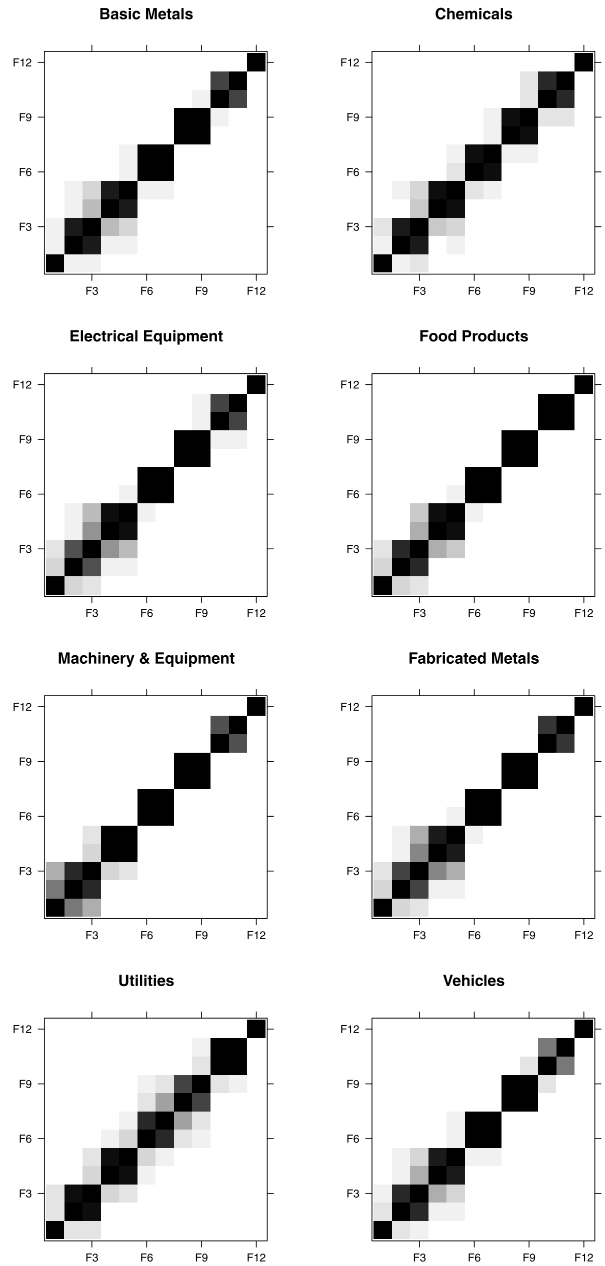

Figure 3. W-correlation matrices of elementary components of the DerivSSA decomposition for eight German IIPs.

4.1 Separability of elementary components

We begin by applying Basic SSA with parameter $L = 12$ to the German IIPs. This particularly small value for L is chosen because IIPs tend to have rather complex trends, which would be very difficult to capture with a large window length.

The quality of the Basic SSA decomposition can be assessed by looking at plots of the L elementary components and the plot of the W-correlation matrix of these components. This matrix shows the level of orthogonality of components and allows for easy identification of whether elementary components are well separated or mixed.

We can see in Figure 2 that the Basic SSA decomposition contains components, many of which are correlated since there are numerous dark grey off-diagonal entries. The darkness of the shade corresponds to a higher degree of correlation, i.e. a lower level of separability. Thus, Basic SSA fails to give a good decomposition with separable, non-mixed components for the German IIPs.

Following this, we apply DerivSSA with $L = 12$ to the German IIPs. Figure 3 depicts the W-correlation matrices for the DerivSSA decomposition and we can observe that the elementary components are almost uncorrelated except for blocks of size 2 by 2. These blocks indicate that the corresponding components are likely to be oscillations with the same frequency, which can be confirmed by plots of the elementary components and their 2D scatterplots [11, Sect. 2.1.5]. Thus, the DerivSSA decomposition allows for an easier grouping of elementary components, with a clearer interpretation. Specifically, we create groups as follows: group 1 contains the first elementary component, group 2 contains the second and third elementary components, group 3 contains the fourth and fifth elementary components, and so on.

Comparing the W-correlation matrices for Basic SSA and DerivSSA decompositions, we conclude that the use of DerivSSA is more beneficial for studying the German IIPs. Note that the W-correlation matrices for French and UK IIPs are rather similar to the German case.

4.2 Frequency estimation and interpretation of grouped components

For an interpretation of components of the grouped DerivSSA decomposition, we apply the function parestimate from the package Rssa in R to these components. In particular, this function provides estimators by the ESPRIT method for the frequency and rate parameters of the set of eigenvectors given, more details can be found in [20, Section 2.4.2.4. and 3.8.2] and [10, 12, 16].

In Table 1 we show estimators of the period of components for the German Food Production IIP. We can observe that estimators of the period are close to values 2, 2.4, 3, 4, 6, and 12 which are usually observed in seasonal data.

In Figure 4 we depict components of the grouped DerivSSA decomposition for the German Food Production IIP, where the last component G7 represents the trend. We can observe that the cyclical behaviour of components G1,…, G6 is rather stable in terms of the period but the amplitude can vary. We also see that the amplitude of components with 2-, 2.4-, 3-, and 4-points per period is smaller than the amplitude of components with 6- and 12-points per period. Since the amplitude of components G5 and G6 is rather stable, we conclude that the single application of DerivSSA to the German Food Production IIP is sufficient, as it resulted in a good decomposition of the original time series into a sum of components with clear interpretation.

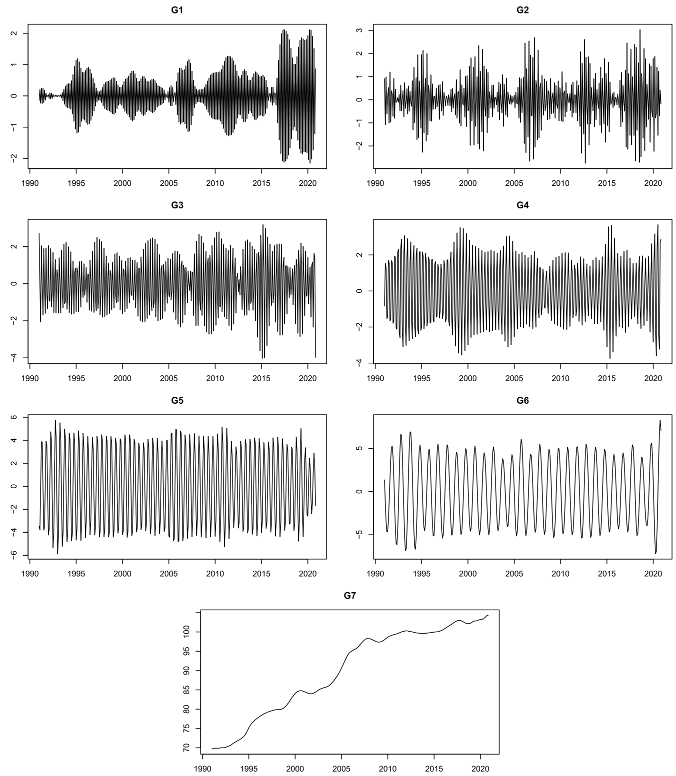

Figure 4. Components of the grouped DerivSSA decomposition for the German Food IIP.

In Table 2 we show estimators of the period of components of the grouped DerivSSA decomposition for the UK Vehicles IIP. Again these components can be interpreted as cycles with approximately 2-, 2.4-, 3-, 4-, 6-, and 12-points per period.

In Figure 5 we observe the components of the grouped DerivSSA decomposition for the UK Vehicles IIP, we can see that these cycles have non-constant amplitude. The 12-month cycle (component G6) is not well extracted because it is poorly separated from the trend, while the 6-month

Table 1. Estimators of the period of components of the grouped DerivSSA decomposition for the German Food IIP

Group	1	2	3	4	5	6
Elementary components	1	2&3	4&5	6&7	8&9	10&11
Estimator of the period	2.000	2.362	2.912	4.004	5.992	12.129

Table 2. Estimators of the period of components of the grouped DerivSSA decomposition for the UK Vehicles IIP

Group	1	2	3	4	5	6
Elementary components	1	2&3	4&5	6&7	8&9	10&11
Estimator of the period	2.000	2.393	2.943	3.988	5.810	11.649

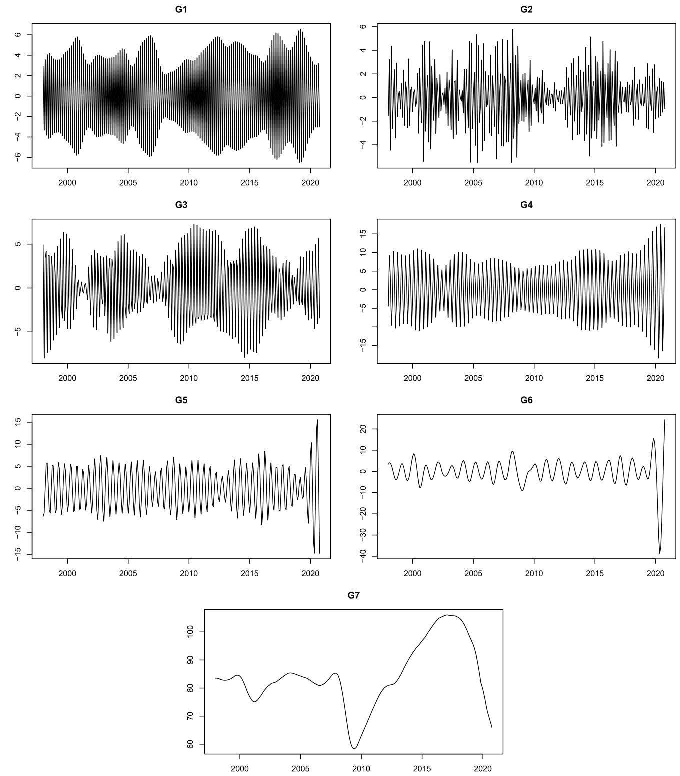

Figure 5. Components of the grouped DerivSSA decomposition for the UK Vehicles IIP.

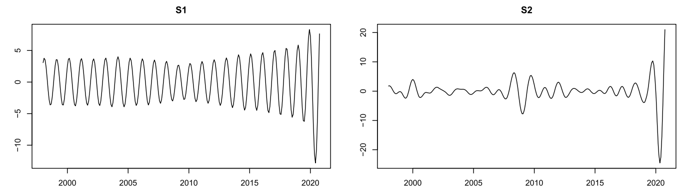

Figure 6. Components of the grouped Basic SSA decomposition for the component G6.

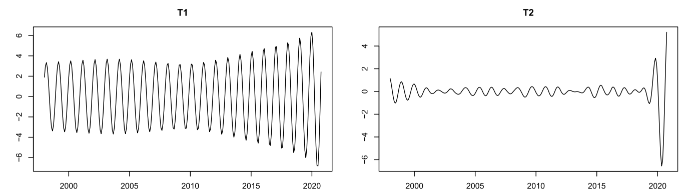

Figure 7. Components of the grouped Basic SSA decomposition for the component S1.

cycle (G5) also suffers from this, although the degree of non-separability is weaker.

Particularly, G6 shows higher volatility corresponding to 2020, when the trend exhibits the essential structural changes. Therefore, we have to decompose the component G6 into the sum of a nice 12-month cycle and a remainder, which can be interpreted as a correction to the trend G7.

4.3 Decomposition of the component G6 for the UK Vehicles IIP

We notice that for the example of the grouped DerivSSA decomposition for the UK Vehicles IIP (where the influence of COVID-19 is particularly harsh), there are one or more components with a non-simple shape, that is, their amplitudes show high disparities caused by outside influence of other components due to incorrect separation and, therefore, require a further decomposition. Since the component G6 has the form of a corrupted 12-month cycle, we have to decompose G6 into the sum of an appropriate 12-month cycle and a remainder, as previously mentioned. This task can be achieved by Basic SSA with large window length. In Figure 6 we show the grouped Basic SSA decomposition with $L = 60$ for the component G6, where the group S1 contains the first and second elementary components and the group S2 contains all other elementary components

The component S1 still has the form of a corrupted 12-month cycle, but there has been some improvement compared to G6, since the degree of corruption is lower, in the sense that the amplitude variation is less volatile. However, there is still a drastic increase in amplitude in 2020 indicating the presence of outside impact from COVID-19 which should be attributed to the trend. Hence, we have to decompose S1 again for a better extraction of the 12-month cycle.

Figure 7 depicts the grouped Basic SSA decomposition with $L = 60$ for the component S1, where the group T1 contains the first and second elementary components and the group T2 contains all other elementary components. We observe that the component T1 has the shape of a nice 12-month cycle without any drastic changes in amplitude. Finally, we consider the components S2 and T2 as a correction to the trend, specifically the trend for the UK Vehicles IIP is the sum of G7, S2, and T2.

For Chemicals, Basic Metals, Fabricated Metals, Machinery and Equipment, and Electrical Equipment IIPs it is sufficient to apply the Basic SSA decomposition to G6 once for obtaining a good 12-month cycle.

5. IMPACT OF COVID-19 ON IIP TRENDS

In Figure 8 we show trends for all IIPs which are obtained by a combination of DerivSSA and Basic SSA decompositions.

For all the series the initial step is to apply DerivSSA with $L = 12$ and $r = 12$. In the DerivSSA decomposition, the components 1 to 9 correspond to subcycles of the seasonality (ordered in decreasing frequency), components 10 and 11 are the 12-month cycle that may need further cleaning, and component 12 corresponds to the trend that may

Figure 8. Trends of IIPs for Germany, France and the UK from 2018 to 2020.

need a correction. For Utilities and Food IIPs, due to the absence of a complex trend, one DerivSSA decomposition is sufficient and no further work is required for a good separation, hence components 1 to 11 can be attributed to the seasonality immediately and component 12 is attributed to the trend. For other IIPs further decomposition is required, hence we have to decompose the corrupted 12-month cycle by applying Basic SSA with $L = 60$. For the Vehicle IIPs for Germany and the UK, we repeated iteratively the Basic SSA decomposition to extract the 12-month cycle, as shown in Section 4.3.

To measure the impact of COVID-19, we compute the decline of the IIP trend as follows

$$D = 100\% \min_{t \in M} \frac{I_t - I_{Oct2019}}{I_{Oct2019}},$$

where I_t is an IIP at month t and M is the set of months in 2020. We present this decline for the 3 countries and 8 IIPs in Table 3. We observe that the declines of different sectors are rather similar to each other across the three countries.

Table 3. The decline of IIPs trends as a percentage change

IIP series	Germany	France	UK
Food Products	-0.1%	-1.2%	-1.3%
Basic Metals	-20.7%	-30.6%	-19.1%
Chemicals	-5.2%	-13.6%	0.3%
Fabricated Metals	-17.4%	-28.6%	-19.5%
Utilities	-4.2%	-3.2%	-1.1%
Machinery&Equipment	-14.2%	-22.1%	-31.0%
Electrical Equipment	-11.9%	-18.1%	-15.1%
Vehicles	-45.5%	-52.7%	-60.2%

As expected, the results in Table 3 corroborate the hypothesis that the industrial production of essential products, such as food and utilities, would be less negatively impacted than that of non-essentials, like vehicles. This is a good indication that the procedure shown has been correctly applied and the trends successfully extracted, allowing for reliable results and inference. Hence, the demonstration can be appropriate for other challenging cases where structural breaks and weak separability are present.

In fact, the decline of the Food IIPs is very small across all countries. This can be explained by the countries' need to maintain domestic food production at a similarly high level to overcome shortages created by the pandemic's effect on the food exports from other countries, coupled with the consumer's unwavering demand for food.

Another inference from these results is that the Chemicals IIPs did not suffer as much as other sectors, in fact, the UK saw an increase during 2020. This may be linked to the type of shock experienced and other specifics of the chemical industry. In particular, since the economic decline was caused by the COVID-19 pandemic, chemicals used in medications or disinfectants also became essential in this case, hence explaining the negligible influence on the industrial production of chemicals. Additionally, pharmaceutical companies from the UK and Germany have actively carried out research and created vaccinations for the virus (e.g. Oxford–AstraZeneca, Pfizer–BioNTech respectively) which heavily rely on certain chemicals and is thus reflected in the results.

For the rest of the IIPs which were quite strongly impacted, there seem to be some differences between countries. For example, the UK experienced an abnormally high fall in Machinery and Equipment IIP in comparison to the other countries, whereas France had a similar, large decline in Basic and Fabricated Metals IIPs. This shows that indeed the influence of COVID-19 has been non-uniform across sectors of production and countries.

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