

Surveys in Differential Geometry

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Volume XIII

Surveys in Differential Geometry

Geometry, Analysis, and Algebraic Geometry:
Forty Years of the Journal of Differential
Geometry

edited by
Huai-Dong Cao and Shing-Tung Yau

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Huai-Dong Cao and Shing-Tung Yau, editors

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Preface

Publication of the Journal of Differential Geometry began in 1967, and for the past 40 years it has flourished, coming to be regarded as the leading journal on the subject.

In 1980, C.-C. Hsiung asked me to be chief editor of the JDG. I was reluctant, for many obstacles presented themselves. However, many great geometers came to the support of the journal. Almost immediately after establishing a new editorial board, we were able to obtain the highly regarded works of Freedman, Donaldson, Schoen, Uhlenbeck, Witten, etc. The editors continue to be deeply grateful for the tremendous support lent by these authors and many others. The quality of the JDG is as strong as ever. (A few years ago, in fact, we found it necessary to increase the journal's size, that we might do justice to the depth and excellence of the submissions received.)

The JDG's 40th anniversary was celebrated at the the Seventh Conference on Geometry and Topology, held at Harvard University in May 2008. The outstanding survey papers making up this volume are the outcome of that conference.

Professor C.-C. Hsiung has just passed away. We would like to dedicate this volume to the memory of him. He was the founding father of the journal, and editor-in-chief these past 40 years. His contribution is immeasurable.

Shing-Tung Yau
Harvard University
June 2009

Contents

<i>Preface</i>	<i>v</i>
Special Lagrangian fibrations, wall-crossing, and mirror symmetry	
Denis Auroux	1
Sphere theorems in geometry	
Simon Brendle and Richard Schoen	49
Geometric Langlands and non-abelian Hodge theory	
R. Donagi and T. Pantev	85
Developments around positive sectional curvature	
Karsten Grove	117
Einstein metrics, four-manifolds, and conformally Kähler geometry	
Claude LeBrun	135
Existence of Faddeev knots in general Hopf dimensions	
Fengbo Hang, Fanghua Lin, and Yisong Yang	149
Milnor K_2 and field homomorphisms	
Fedor Bogomolov and Yuri Tschinkel	223
Arakelov inequalities	
Eckart Viehweg	245
A survey of Calabi-Yau manifolds	
Shing-Tung Yau	277