Frontiers of Mathematical Sciences

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The Inauguration of the Mathematical Sciences Center of Tsinghua University and the Tsinghua-Sanya International Mathematics Forum

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Frontiers of Mathematical Sciences:
The Inauguration of the Mathematical Sciences Center of Tsinghua University
and the Tsinghua-Sanya International Mathematics Forum

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Poem: On the Tsinghua University Centennial Celebration

Shing-Tung Yau

清華大學百年誌慶

丘成桐

懷厚德自強之心胸，開天闢地；

秉自由獨立之情操，尋美求真。

Preface

The Editorial Board

Tsinghua University is arguably a leading university in China, with a glorious history, and it is marching towards the goal of becoming one of the first-class universities of the world. In this process, mathematics has played and will continue to play an essential role. The purpose of this book is to present—from a global perspective—the university's past achievements, exciting new developments, and future plans for mathematics. The Mathematics Department at Tsinghua University was the cradle of modern mathematics in China. Together with the Mathematical Sciences Center and the Tsinghua-Sanya International Mathematics Forum, it is leading Chinese mathematics to a new, higher stage of development.

Many distinguished Chinese mathematicians have studied and taught at Tsinghua University. They include Shiing-Shen Chern, Loo-Keng Hua, Chia-Chiao Lin, Pao-Lu Hsu, whose works have had a lasting impact on mathematics. One purpose of this book is to illustrate their lives and to explain their scientific contributions to readers around the world. Tsinghua University has always emphasized international scientific exchange, and many world-famous mathematicians such as Norbert Wiener and Jacques Hadamard have visited and taught at Tsinghua University. We wish to take this opportunity to describe such mathematicians' visits and their interactions with the young Chinese mathematicians at Tsinghua, and thus to express our appreciation for their help.

The newly established Mathematical Sciences Center at Tsinghua University has likewise invited many internationally known mathematicians from around the world to teach and work here: we expect that their visits will have a similarly profound impact on the next generation of Chinese mathematicians, and thereby contribute to the further development of Chinese mathematics.

Besides the discussion of mathematics at Tsinghua, this book introduces the new Tsinghua-Sanya International Mathematics Forum. Many short-term conferences on diverse topics in mathematical sciences at the highest level will be held here, and the forum is certain to become one of the most sought-after platforms on which mathematicians from around the world gather to exchange ideas and share new results. To the inaugural conference held in December 2010, we invited many first-class mathematicians and physicists to give accessible talks on the most recent breakthroughs at the frontiers of mathematical sciences, and we are truly honored to have their participation. Besides expanded lecture notes from most of the talks given at this conference, this book also presents the greetings

and speeches given there by the presidents of several major universities, and by leaders from the central government and local governments of China.

The publication of this book is the fruit of a collaboration of many people. Without their help and dedication, this book could not have been edited and published in such a short time. First, we would like to thank all the people who have contributed papers and articles. Secondly, we wish to thank Ms. Yan Zhang for keeping track of all correspondences in the process of editing this book, Professor Yunbo Zeng for coordinating the typesetting of the papers, and Professor Yuefei Wang for translating articles from Chinese for this English version of the book. We would also like to thank Mr. Lixin Qin of International Press for coordinating efficiently publication of both versions of this book, and Mr. Brian Bianchini of International Press for his devoted help with typesetting and editing of both books. Finally, we wish to thank the staff of the Mathematical Sciences Center at Tsinghua University—Ms. Jiangnan Zhao, Ms. Lei Zhang, Ms. Fei Li, and Ms. Lijuan Sun—for their help during the preparation of both versions of this book.

The Editorial Board:
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Preface

Binglin Gu
President of Tsinghua University

As queen of sciences, mathematics has been applied to, and has interacted with, natural sciences, engineering sciences, and social sciences, and has played an important and fundamental strategic role in the growth and development of a university in terms of general education, cultivation of tip-top talents, and scientific values of outcomes of original researches on the cutting edge of studies of mathematics.

The discipline of mathematics has been developed in Tsinghua University over a long period of time and has yielded remarkable fruits. Since the founding of the Mathematical Sciences Department at Tsinghua University in 1927, many mathematicians well-known both in China and abroad—such as Zhifan Zheng, Qinglai Xiong, Wuzhi Yang, Xuefu Duan, Loo-Keng Hua, Shiing-Shen Chern, and Pao-lu Hsu—have been cultivated. After the reorganization of universities and departments in the 1950s, Tsinghua University became a university focusing on engineering education. However, mathematics was taught as a basic course, and relevant research of mathematics in combination with engineering disciplines has never come to a halt. Since China's adoption of the policy of reform and opening to the outside world, the Mathematical Sciences Department has achieved a rapid recovery and made great strides forward in construction of discipline.

Professor Chia-Chiao Lin, a famous mathematician, returned to Tsinghua University in 2002, establishing the Peiyuan Zhou Applied Mathematics Research Center and starting an international forum about issues on the cutting edge of applied mathematics. In recent years, Tsinghua University has been strengthening international exchanges and cooperation, particularly in the field of mathematics, including a joint cultivation of talents with Université Paris XI, inviting lecturers and professors from famous universities in the United States and France, and sending outstanding students and teachers to learn from well-known teachers in the field of mathematics throughout the world, and has achieved positive outcomes.

In 2009, with tremendous support from Professor Shing-Tung Yau, the Mathematical Sciences Center of Tsinghua University was founded and commenced operation as an "academic special zone"—another milestone in the development of the discipline of mathematics of Tsinghua University and a significant strategy of Tsinghua University in establishing itself as one of the top universities in the world. In the course of only about one year, under the leadership of Professor Yau, the Center has made admirable accomplishments, with the invitation many top senior mathematicians, outstanding young

mathematicians, and the most extraordinary mathematicians across the world, to teach at the Center.

Last year, Professor Yau proposed establishment of an international forum of mathematics to provide support to the Center in its development into a premier center for mathematical research. This proposal was highly appreciated by senior officials from the central government, and garnered the support of the secretary of the party committee of Hainan Province Liucheng Wei, the governor of Hainan Province Baoming Luo, the secretary of the party committee of Sanya, Zelin Jiang, and generous financial support from an alumnus, Mr. Weixue Shi. The First Tsinghua-Sanya International Mathematics Forum was held in December 2010 in the beautiful coastal city of Sanya, with the attendance of many internationally famous scholars and leaders from universities who delivered impressive speeches and presented high-level academic reports. We have compiled this book by collecting those speeches and academic reports as a summary of the work of the Center over the past year and a half, and as a precious gift on the occasion of celebration of the 100^{th} anniversary of the founding of Tsinghua University.

We have confidence that the Mathematical Sciences Center of Tsinghua University will distinguish itself as one of top centers of mathematics research and talent cultivation in the world under the leadership of Professor Yau, together with the assistance and support of people from all walks of life, and that the Center shall make its due contribution to the development of the Chinese nation and to the development of sciences for mankind.

Preface to the Sanya Forum

Shing-Tung Yau

Mathematics has come a long way since our ancestors began to count by making nodes in rope and with sticks, but its essence stays the same: mathematics is the language of nature, and it is the queen and servant of science.

As we know, mathematical research is becoming more and more a collective effort in exchanging and sharing of ideas, which is essential for its advancement. To promote mathematical research, we need not only to provide good working environments for mathematicians to focus on their research, but also to provide them a platform to exchange ideas on new directions and new methods. We also know that besides supporting long-term postdoctoral fellows and faculties, more frequent short-term interactions between scientists of different disciplines have been proved to be crucial for this purpose.

To achieve this purpose, many centers have been established successfully in the West in the past fifty years, but no such platform has been found in Asia. Fortunately, with the strong support of the Hainan Provincial Government, the Sanya City Government, and Tsinghua University, an agreement was signed last June to establish the Tsinghua-Sanya International Mathematics Forum in this beautiful coastal city of Sanya, on the Southern China Sea. Establishing this forum will mean a large-scale interaction between theoretical and applied scientists of different disciplines from all over the world. While we will focus largely on fields related to mathematics, we will not exclude activities of some interesting but more speculative branches of theoretical sciences. Establishment of the Tsinghua-Sanya International Mathematics Forum will be remembered as an important historical event in mathematical science.

For decades, I have been learning from, conducting, and promoting mathematical research. I learned most of my advanced mathematics in the United States, and I am grateful to my teachers and friends in the States. It is a great country that helps to build up modern mathematical science, but nevertheless many great mathematicians in the U.S. have come from all over the world. I believe it is also a duty of the U.S. to help other countries in promoting this beautiful subject. Research and advancement in the discipline of mathematical science clearly should be a world-wide effort. We have seen many such contributions made by many countries towards China. Besides our U.S. colleagues, our French colleagues have come in the past thirty years to help Wuhan University, Fudan University, Tsinghua University, etc. Many outstanding young Chinese mathematicians have been trained by them. We also have witnessed good contributions made by our colleagues from the United Kingdom, Germany, Japan, Israel, and Korea, and many other countries.

Over the past thirty years, I have spent a lot of time and effort to help to raise the level of mathematical research in China. I not only was born in China and love China, but I also see the immense potential of young mathematicians in this region, and I have been working to build some proper environment in which young mathematicians can prosper over the years. I am thankful for the support of the Chinese government and the University leaders, as well as for the support of many fellow mathematicians. I have built several mathematical research centers and lead many important mathematics education activities at both high-school and college level. Last December, with the strong support of Tsinghua University, the Mathematical Sciences Center (MSC) of Tsinghua University was established. I am pleased to report that, in one short year, we have already recruited several young and promising mathematicians to the Center.

I have also been working to establish a world-class international conference center that will be truly useful for scientists in exchanging and creating new ideas. I envision an ideal environment that should not be interfered with, or distracted, by any unhelpful or non-research events. I see such a world-class center as an open mathematics forum to facilitate the world's leading mathematicians in exploring new directions, and in developing new methods by interacting within a small group of scientists in the fields of pure mathematics, applied mathematics, statistics, theoretical and applied physics, theoretical biology, or other disciplines of importance. Since the Center is big enough to run two or three programs at the same time, mathematicians can interact with physicists, biologists and engineers in a way that is difficult to achieve even in universities, where professors are generally occupied with teaching, services, and other administrative duties, besides their research. That is also why I want the Tsinghua-Sanya International Mathematics Forum (TSIMF) to be conceived as a unique working environment dedicated solely to research and creativity. Mathematicians and other scientists come here for only one thing: to think the unthinkable, to try the untried, to make the impossible possible. In short, to interact and create. I have good reason to expect great ideas be born right here in years to come; I also look forward to seeing scientists in mathematics and other fields feeling proud to present their new findings in this Center.

The inauguration conference was the first activity of the TSIMF timetable. The speakers and participants consisted of world-class mathematicians, physicists, statisticians, presidents of premier universities, and directors of research institutes from all over the world—prefiguring the high academic standards and international nature to be expected of future activities of the TSIMF. For this, I am most grateful to all the speakers and participants, especially those that have come from far away: I will count on your continue support in the future development of TSIMF.

Due to the time constraint, the papers of several speakers could not be prepared for publication in time for inclusion in this book. Among those, I would like to mention the talk by David Gross, a Nobel laureate in physics. The title of his talk was "Physics and Mathematics at the Frontier." It gives a beautiful survey of inseparable interaction between mathematics and physics. Professor Gross has asked for some time to think further about this excellent paper, and we do regret that, therefore, it is not yet ready for publication in this volume.

Both Robbert Dijkgraaf and Hirosi Ooguri presented beautiful expositions which demonstrate clearly the close and fruitful interaction, over the past twenty-five years, of the physics of string theory with mathematics. Jun Li's paper showed how powerful algebraic geometric methods can be used to understand the mathematics inspired by the un-

derstanding of string theory, and Ib Madsen discussed the important and classical question of the topology of moduli problem, where he himself has made fundamental contributions.

Wilfried Schmid and Kari Vilonen offered general conjectures on unitary dual of reductive Lie groups, by applying Hodge theory. Benedict Gross related the Bhargava's representation theory with Vinberg's invariant theory, while Laurent Fargues and Jean-Marc Fontaine studied factorization of analytic functions in mixed characteristic, which is linked to p-adic Hodge theory. In analysis, Chang-Shou Lin studied topological methods in partial differential equations. Zhouping Xin explored the very difficult problem of Navier-Stokes equations, Ker-Chau Li applied statistics methods to genomic data analysis, and Jun Liu and his co-authors applied statistics to the very interesting topic of mining Chinese historical documents.

Finally, I would like to take this opportunity to express my gratitude for the great vision and vital support of the leaders of the Hainan Provincial Government, especially that of Party Secretary Wei and Governor Luo, the leaders of Sanya City Government, and the leaders of Tsinghua University. I owe special thanks to Ms. Yuqun Hao and Mr. Zhelin Jiang for their tireless effort and constant support in making this conference a great success.