Morningside Lectures in Mathematics
Volume I

Lectures on the Analysis of Nonlinear Partial Differential Equations: Part 1

edited by

Fanghua Lin
Xueping Wang
Ping Zhang

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MORNINGSIDE LECTURES IN MATHEMATICS

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Preface

In summer of 2001, we initiated a summer school program on the harmonic analysis and its applications in nonlinear partial differential equations, with special emphases on nonlinear Schrodinger equations, kinetic equations of Bolzmann type and classical fluid equations. Over the years, there have been many distinguished mathematicians working in these fields who have come to help our program and to give series of special lectures. The program has been shown to be particularly helpful to young researchers and students. The lectures involved have gradually turned into more formal and regular seminars on *Analysis in Partial Differential Equations* at the Morningside Center of Mathematics Academy of Mathematics and System Science, Chinese Academy of Sciences.

From June 2007 to January 2008, we held a special semester PDE– program. We invited many mathematicians and experts in mathematical theory of fluid mechanics and quantum mechanics. The visitors during that period includes: Bresch Didier, Carles Rémi, Jean-Yves Chemin, Desvillettes Laurent, Lopes Filho Milton C, Nussenzveig Lopes Helena J. Novotny, Antonin, Chao-Jiang Xu, Chongchun Zeng, Ping Zhang and Yuxi Zheng, who gave a series of lectures and provided excellent lecture notes. It is no doubt that these lecture notes would be very useful for many researchers and students. In this volume, we have collected lecture notes by M. C. Lopes concerning the boundary layers of incompressible fluid flow; by C. J. Xu on the micro-local analysis and its applications to the regularities of kinetic equations; by Y. X. Zheng on the weak solutions of variational wave equation from liquid crystals, and by P. Zhang and Z. F. Zhang on the free boundary problem of Euler equations. In addition,we also included the notes by F. Nier on the hypoellipticity of Fokker-Planck operator and Witten-Laplace operator that were given earlier in the summer of 2006.

We have planned to publish in the forthcoming volumes the other lectures notes. Some are from past lectures at our program and some will be collected from the newly scheduled seminars. We hope that the publication of these lecture notes may provide valuable references and up-to-date descriptions of current developments of various related research topics, that will benefit many young researchers or graduate students. We wish to take this opportunity to thank the Morningside Center of Mathematics, the Institute of Mathematics of AMSS that

provides all necessary supports. We are particularly grateful to professor Lo Yang for his constant help, supports and encouragements to our program. We also would like to thank Guilong Gui for his careful preparations of the latex file of the entire book. We finally appreciate for the financial support from the Chinese Academy of Sciences.

Fanghua Lin in New York
Xueping Wang in Nantes
Ping Zhang in Beijing
On November 3, 2008

Contents

Milton C. Lopes Filho: Boundary Layers and the Vanishing Viscosity Limit for Incompressible 2D Flow 1

Francis Nier: Hypoellipticity for Fokker-Planck Operators and Witten Laplacians 31

Chaojiang Xu: Fourier Analysis of Non Cutoff Boltzmann Equations 85

Ping Zhang and Zhifei Zhang: On the Free Boundary Problem of 3-D Incompressible Euler Equations 199

Yuxi Zheng: An Introduction to a Variational Wave Equation 285