

Surveys of Modern Mathematics
Volume II

Lie Theory and Representation Theory

edited by

Naihong Hu
Bin Shu
Jianpan Wang

 International Press
www.intlpress.com

 高等教育出版社
HIGHER EDUCATION PRESS

Surveys of Modern Mathematics, Volume II
Lie Theory and Representation Theory

Naihong Hu
Bin Shu
Jianpan Wang

2010 Mathematics Subject Classification. 16T05, 14F10, 16G60, 14L15, 17B10, 17B50, 20J06, 20G05, 20G10, 32C38.

Copyright © 2010, 2012 by International Press, Somerville, Massachusetts, U.S.A., and by Higher Education Press, Beijing, China.

This work is published and sold in China exclusively by Higher Education Press of China.

All rights reserved. Individual readers of this publication, and non-profit libraries acting for them, are permitted to make fair use of the material, such as to copy a chapter for use in teaching or research. Permission is granted to quote brief passages from this publication in reviews, provided the customary acknowledgement of the source is given. Republication, systematic copying, or mass reproduction of any material in this publication is permitted only under license from International Press. Excluded from these provisions is material in articles to which the author holds the copyright. (If the author holds copyright, notice of this will be given with the article.) In such cases, requests for permission to use or reprint should be addressed directly to the author.

ISBN: 978-1-57146-236-7

Printed in the United States of America.

16 15 14 13 12 1 2 3 4 5 6 7 8 9

SURVEYS OF MODERN MATHEMATICS

Series Editors

Shing-Tung Yau
Harvard University
Cambridge, Massachusetts
U.S.A.

Lizhen Ji
University of Michigan, Ann Arbor
U.S.A.

Yat-Sun Poon
University of California at Riverside
U.S.A.

Jie Xiao
Tsinghua University
Beijing, China

Jean-Pierre Demailly
Institut Fourier
Laboratoire de Mathématiques
Saint-Martin d'Hères, France

Eduard J.N. Looijenga
Universiteit Utrecht
The Netherlands

Neil Trudinger
Mathematical Sciences Institute
Australian National University
Canberra, Australia

Preface

During the period from July 13 to July 31, 2009, East China Normal University hosted the second workshop and summer school on Lie Theory and Representation Theory. This volume contains the lecture notes of three courses in that summer school, together with the lecture notes of one course given in the first summer school which was held in 2006.

This volume consists of articles by Shun-Jen Cheng and Weiqiang Wang, Rolf Farnsteiner, Daniel K. Nakano, and Toshiyuki Tanisaki. These articles focus on different areas in Lie theory and representation theory. The article jointly by Cheng and Wang introduces some recent developments of representations of Lie superalgebras, explaining how Lie superalgebras of types \mathfrak{gl} and \mathfrak{osp} provide a natural framework for generalized Schur and Howe dualities, and how a super duality gives a conceptual solution to the irreducible character problem for these Lie superalgebras in terms of the classical Kazhdan-Lusztig polynomials.

Farnsteiner's article discusses combinatorial and geometric aspects of representation theory of finite group schemes, and focuses on the "classical" theory of co-commutative Hopf algebras, the defining algebras of affine algebraic group schemes.

Nakano's article gives a survey of recent developments in the representation theory and cohomology theory of reductive algebraic groups, their Frobenius kernels and their associated finite groups of Lie type.

Tanisaki's article presents an overview of the theory of D -modules and its application to representations of Lie algebras.

This volume is well suited for graduate students in the fields of Lie theory and representation theory and related topics, and also for researchers who wish to learn about some current core areas in Lie theory and representation theory and their applications.

At last, we sincerely express our thanks to the Department of Mathematics, the International Exchange Division and the Graduate School of East China Normal University for their financial support to the summer schools and workshops in 2006 and 2009. We are grateful to National Natural Science Foundation of China

Preface

for financial support (Grant:10926022) in 2009. Our deep appreciation also goes to our colleagues Pei Gu, Youyi Wu and Hongyan Zhang for their assistance in organizing these activities.

Jianpan Wang

Bin Shu

Naihong Hu

In Shanghai

31 October, 2010

Contents

<i>Shun-Jen Cheng and Weiqiang Wang: Dualities for Lie Superalgebras</i>	1
0 Introduction	1
1 Lie superalgebra ABC	5
2 Finite-dimensional modules of Lie superalgebras	11
3 Schur-Sergeev duality	18
4 Howe duality for Lie superalgebras of type \mathfrak{gl}	25
5 Howe duality for Lie superalgebras of type \mathfrak{osp}	29
6 Super duality	35
References	42
 <i>Rolf Farnsteiner: Combinatorial and Geometric Aspects of the</i>	
Representation Theory of Finite Group Schemes	47
0 Introduction	47
1 Finite group schemes	49
2 Complexity and representation type	78
3 Support varieties and support spaces	87
4 Varieties of tori	106
5 Quivers and path algebras	114
6 Representation-finite and tame group schemes	125
References	144
 <i>Daniel K. Nakano: Cohomology of Algebraic Groups, Finite Groups,</i>	
and Lie Algebras: Interactions and Connections	151
1 Overview	151
2 Representation theory	153
3 Homological algebra	156
4 Relating support varieties	158
5 Relating cohomology	163
6 Computing cohomology for finite groups of Lie type	168
References	173
 <i>Toshiyuki Tanisaki: D-modules and Representation Theory</i>	177
1 Motivation	177
2 Basic concepts	183

Contents

3	Derived category	189
4	Coherent D -modules	196
5	Regular holonomic D -modules	208
6	Application to representation theory	214
	References	218