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Advanced Lectures in Mathematics  
Volume XXVI

# Handbook of Moduli

## Volume III

edited by

Gavril Farkas · Ian Morrison

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*The Handbook of Moduli is dedicated to the memory of Eckart Viehweg, whose untimely death precluded a planned contribution, and to David Mumford, who first proposed the project, for all that they both did to nurture its subject; and to Angela Ortega and Jane Reynolds for everything that they do to sustain its editors.*





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