Surveys of Modern Mathematics
Volume V

Isolated Singular Points on Complete Intersections

Second Edition

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Surveys of Modern Mathematics

To Elisabeth

Preface to the Second Edition

Almost three decades have passed since this monograph saw the light of day and in the intervening years. Its subject matter has not only matured, but also spread out in new directions. Yet this book is still often used as a reference and so when Higher Education Press and International Press offered me to produce a TEX source file from the original type script that I could revise, I happily accepted. That immediately faced me with the question to what length I should go with bringing it up to date. A proper job would have meant writing another book, an idea I quickly discarded is not just for reasons of time, but also for lack of energy, if not competence. Going halfway did not appeal to me either, and so in the end I decided to stick to the contents of the first edition and confine myself essentially to matters of presentation. This occasionally led to simplifications of proofs, slightly stronger statements, rearrangement of the material, minor additions and, I hope, clearer exposition. There are also some—fortunately modest—changes of notation. I had to make some minor corrections as well and pray that the conversion of a typescript into an edited TEX file did not introduce a new set of errors.

Given the rather restricted purpose of the original list of references and the now common availability of forward search tools as provided by the online databases MathSciNet and ZMATH, I refrained from adding new items, except for some conference proceedings. Of course full reference data were supplied for papers that were announced as to appear in the original edition.

E. Looijenga
Utrecht, August 2012

Preface to the First Edition

In the spring term of 1980 I gave a course on singularities at Yale University (while supported by NSF grant MCS 7905018), which provided the basis of a set of notes prepared for the first two years of the Singularity Intercity Seminar (1980—1982, at Leiden, Nijmegen and Utrecht, jointly run with Dirk Siersma and Joseph Steenbrink). These notes developed into the present book. As a consequence, the aim and prerequisites of the seminar and this book are almost identical.

The purpose of the seminar was to introduce its participants to isolated singularities of complex spaces with particular emphasis on complete intersection singularities. When started we felt that no suitable account was available on which our seminar could be based, so it was decided that I should supply notes, to be used by both the lecturers (in preparing their talks) and the audience. This was quite a purifying process: many errors and inaccuracies of the first draft were thus detected (and often corrected).

The prerequisites consisted of some algebraic and analytic geometry (roughly covering the contents of the books of Mumford (1976) and Narasimhan (1966)), some algebraic topology (as in Spanier (1966) and Godement (1958)) and some facts concerning Stein spaces. Given this background, my goal was to prove every assertion in the text. This has been achieved except for the coherence theorem (8.7) and some assertions in the descriptive Chapter 1. An exception should also be made for the paragraphs marked with an asterisk (*): they generally give useful information which however is not indispensable for what follows and so may be skipped. Perhaps the whole first chapter could have been marked this way. It gives interesting examples of isolated singularities (or of constructions thereof) with the purpose to indicate the position of complete intersection singularities among them and to describe material to which the theory is going to apply. It is mainly for the latter reason that this chapter should not be skipped entirely.

As each chapter has its own introduction, I shall not review the chapters separately, nor the whole book. I believe that the first seven chapters (with the exception of Section 5.C) can be used as a basis for a course on the subject, assuming the audience has approximately the background mentioned above. The contents of Section 5.C and the last two chapters are somewhat more advanced and moreover Chapter 9 is of more specialized nature. Some of the results may be new (at least do not appear in this form in the literature), examples are the discussion (1.25), parts of Chapter 4 (Theorem (4.15) and Corollary (4.11)), the monodromy theorem Section 5.C, (6.17), the variation extension (7.17)–(7.19), and the Sections 9.A and 9.C. The references at the end should be regarded as a list of sources I consulted and not as a bibliography which pretends to be complete in any respect.

Although all the sources I used are cited, I want to single out some papers which were particularly useful for me: Lamotke (1975) for Chapter 3, Teissier (1976) for Chapter 4, Lê (1973, 1978) for Chapter 5 and Greuel (1975, 1980) for Chapters 8 and 9. As I mentioned, the book also benefitted from criticism of the lecturers in the seminar. I mention in particular, C. Cox, W. Janssen, P. Lemmens, P. Lorist, F. Menting, G. Pellikaan, J. Stevens, D. van Straten and E. van Wijngaarden. Also, comments from my co-organizers, Dirk Siersma and Jozef Steenbrink, were very helpful. I take the occasion to thank the Dutch Organization for the Advancement of Pure research (ZWO) for sponsoring the seminar and for supporting three of its participants. I am greatly indebted to W. Janssen for careful proofreading—his accurate job eliminated many errors and obscurities—and help with the exposition. I thank Ms. Ellen van Eldik for producing a beautiful camera-ready typescript. Finally, I express my thanks to my wife, Elisabeth, for her continuous support during the writing of this book.

E. Looijenga
Nijmegen, August 1983

A Few Notational Conventions

If $f : (X, \mathcal{O}_X) \to (Y, \mathcal{O}_Y)$ is a morphism between ringed spaces, then by definition we have a homomorphism $f^{-1}\mathcal{O}_Y \to \mathcal{O}_X$ of sheaves of rings on X, an observation we only make in order to show our notation for a sheaf pull-back. Hence, if $\mathcal{F}$ is an $\mathcal{O}_Y$-module, then $f^{-1}\mathcal{F}$ stands for its ordinary sheaf pull-back (so that it will be a sheaf of $f^{-1}\mathcal{O}_Y$-modules), but we shall reserve the notation $f^*\mathcal{F}$ for $\mathcal{O}_X \otimes_{f^{-1}\mathcal{O}_Y} f^{-1}\mathcal{F}$. Notational logic dictates that we then denote $f^{-1}\mathcal{O}_Y \to \mathcal{O}_X$ on stalks likewise: $f_x^{-1} : \mathcal{O}_{Y,f(y)} \to \mathcal{O}_{X,x}$, but we dare not deviate too much from convention and write $f_x^*\phi \in \mathcal{O}_{X,x}$ for the image of $\phi \in \mathcal{O}_{Y,f(y)}$ under this ring homomorphism.

As a rule we use for natural pairings (such as between a vector space and its dual) what physicists call the bra-ket notation, $\langle \mid \rangle$, whereas a bilinear form is often denoted by a centered dot separating the variables: $(\cdot)$.

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