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Volume 30

Automorphic Forms and *L*-functions

edited by

Jianya Liu

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Automorphic Forms and L -functions

Volume Editor:

Jianya Liu (School of Mathematics, Shandong University)

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Preface

This is a collection of lecture notes from the CIMPA-UNESCO-CHINA Research School 2010: Automorphic Forms and L -functions, held at Shandong University at Weihai, in August 1–14, 2010.

The scientific directors were W. Duke and P. Sarnak (Chair) who provided detailed advice regarding the important aspects of the research school. The invited speakers were J. Cogdell, G. Harcos, E. Lapid, Xiaoqing Li, Wenzhi Luo, P. Michel, A. Reznikov, F. Shahidi, and Yangbo Ye.

Each of the speakers was scheduled to give a series of four lectures. Shahidi, although could not come to Weihai in person, has given full support to the research school by making his notes available, and Cogdell expertly combined his own lecture notes with Shahidi's material to give eight lectures. Not only have the speakers given excellent lectures, but they have also spent time completing their lecture notes – available for us here. Besides the speakers, there were fifteen scholars from all over the world plus ten international graduate students and over sixty Chinese graduate students who participated in various activities offered by the research school. The diligence and enthusiasm, professional skills and hard work of speakers, participants and organizers have contributed to the great success of the research school.

M. Waldschmidt visited Weihai in 2007 and provided clear instructions on how to organize a CIMPA school. M. Jambu, representative of CIMPA, not only participated the whole school, but also conducted lots of administrative duties on behalf of CIMPA before and after the school.

It is a great pleasure to express my sincere thanks to all the friends who have made this research school possible and who have contributed to its success. Thanks are also due to Lizhen Ji, Liping Wang, and Huaying Li for their effort and patience, and to Taiyu Li for helping me prepare the TeX file for publication. This event would not have been possible without the financial support from Shandong University, the NSFC, and CIMPA-UNESCO, and I would like to express my gratitude and thanks to them.

Jianya Liu
School Coordinator
July 2013

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The CIMPA-UNESCO-CHINA Research School 2010: Automorphic Forms and L -functions

Contents

<i>L</i>-functions and Functoriality	1
<i>James W. Cogdell</i>	
I <i>L</i> -functions for $GL(n)$ and Converse Theorems	
1 Modular Forms and Automorphic Representations	3
2 <i>L</i> -functions for GL_n and Converse Theorems	8
II <i>L</i> -functions via Eisenstein Series	
3 The Origins: Langlands	15
4 The Method: Langlands-Shahidi	21
5 The Results: Shahidi	26
III Functoriality	
6 Langlands Conjectures and Functoriality	30
7 The Converse Theorem and Functoriality	36
8 Symmetric Powers and Applications	41
References	47
Twisted Hilbert Modular <i>L</i>-functions and Spectral Theory	49
<i>Gergely Harcos</i>	
1 Lecture One: Some Quadratic Forms	49
2 Lecture Two: More Quadratic Forms	53
3 Lecture Three: Preliminaries from Number Theory	56
4 Lecture Four: Subconvexity of Twisted <i>L</i> -functions	61
Acknowledgments	64
References	64
The Voronoi Formula for the Triple Divisor Function	69
<i>Xiaoqing Li</i>	
1 Introduction	69
2 Proof of the Main Theorem	71
Acknowledgments	89
References	89

Linnik's Ergodic Method and the Hasse Principle for Ternary Quadratic Forms	91
<i>Philippe Michel</i>	
1 Foreword	91
2 Integral Quadratic Forms	92
3 The Hasse Principle	93
4 Quadratic Forms over Lattices	96
5 Equidistribution on Adelic Quotient	101
6 Properties of the Adeles	106
7 The Hasse Integral Principle and Equidistribution of Adelic Orbits	114
8 The Ergodic Method	118
References	129
Automorphic Periods and Representation Theory	131
<i>Andre Reznikov</i>	
1 Automorphic Representations and Frobenius Reciprocity	131
2 Bounds on Periods and Representation Theory	143
Acknowledgments	147
References	147
Eisenstein Series, L-functions and Representation Theory	149
<i>Freydoon Shahidi</i>	
1 Preliminaries	150
2 L -Groups, L -Functions and Generic Representations	151
3 Eisenstein Series and Intertwining Operators; The Constant Term	154
4 Constant Term and Automorphic L -Functions	156
5 Examples	160
6 Local Coefficients, Nonconstant Term and the Crude Functional Equation	160
7 The Main Induction, Functional Equations and Multiplicativity	162
8 Twists by Highly Ramified Characters, Holomorphy and Boundedness	166
9 Examples of Functoriality with Applications	168

10 Applications to Representation Theory	171
References	173
Lecture Notes on Some Analytic Properties of Automorphic	
L-functions for $SL_2(\mathbb{Z})$	179
<i>Yangbo Ye</i>	
1 Introduction	179
2 An Integral Representation and Functional Equation	180
3 A Converse Theorem	188
4 The Phragmén-Lindelöf Principle and Convexity	193
5 The Rankin-Selberg Theory	197
References	202