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Gökova Geometry-Topology Conference 2015

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PREFACE

This year, in Gökova, we had many interesting talks on high dimensional geometric, symplectic and contact topology, as well as on low-dimensional manifolds. We also had two very stimulating mini-courses, one on the triangulation theorem, by Ciprian Manolescu; and the other on the higher dimensional contact geometry by Emmy Murphy. We thank all the participants for making this conference a very informative enjoyable event. We thank TMD (Turkish Mathematical Society), and NSF (National Science Foundation) for funding this conference, and thank International Press for printing and distributing these proceedings. We finally thank Hotel Yücelen (which is located on the scenic shores of the Gökova Bay) for supporting and hosting this conference.

March 2016 The Editors

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Lectures on the triangulation conjecture

Ciprian Manolescu

Abstract. We outline the proof that non-triangulable manifolds exist in any dimension bigger than four. The arguments involve homology cobordism invariants coming from the Pin(2) symmetry of the Seiberg-Witten equations. We also explore a related construction, of an involutive version of Heegaard Floer homology.

1. Introduction

The triangulation conjecture stated that every topological manifold can be triangulated. The work of Casson [1] in the 1980's provided counterexamples in dimension 4. The main purpose of these notes is to describe the proof of the following theorem.

Theorem 1.1 ([26]). *There exist non-triangulable n-dimensional topological manifolds for every $n \geqslant 5$.*

The proof relies on previous work by Galewski-Stern [15] and Matumoto [27], who reduced this problem to a different one, about the homology cobordism group in three dimensions. Homology cobordism can be explored using the techniques of gauge theory, as was done, for example, by Fintushel and Stern [9, 10], Furuta [13], and Frøyshov [12]. In [26], Pin(2)-equivariant Seiberg-Witten Floer homology is used to construct three new invariants of homology cobordism, called α, β and γ. The properties of β suffice to answer the question raised by Galewski-Stern and Matumoto, and thus prove Theorem 1.1.

The paper is organized as follows.

Section 2 contains background material about triangulating manifolds. In particular, we sketch the arguments of Galewski-Stern and Matumoto that reduced Theorem 1.1 to a problem about homology cobordism.

In Section 3 we introduce the Seiberg-Witten equations, finite dimensional approximation, and the Conley index. Using these ingredients, we review the construction of Seiberg-Witten Floer stable homotopy types, following [25].

In Section 4 we explore the module structure on Borel homology, and more specifically on the Pin(2)-equivariant homology of the Seiberg-Witten Floer stable homotopy type. Using this module structure, we define the three numerical invariants α, β, γ, and show that they are preserved by homology cobordism.

Key words and phrases. Triangulations, Seiberg-Witten equations, Floer homology.
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Section 5 contains material about equivariant Spanier-Whitehead duality. This is applied to understanding the behavior of α, β, γ under orientation reversal. Showing that $\beta(-Y) = -\beta(Y)$ completes the proof of Theorem 1.1.

In Section 6 we outline the construction of involutive Heegaard Floer homology, joint work of Hendricks and the author [18]. Involutive Heegaard Floer homology is a more computable counterpart to $\mathbb{Z}/4$-equivariant Seiberg-Witten Floer homology, and has its own applications to questions about homology cobordism.

2. Triangulations

2.1. Basic definitions

A *triangulation* of a topological space X is a homeomorphism from X to a simplicial complex. Let us recall that a *simplicial complex* K is specified by a finite set of vertices V and a finite set of simplices $S \subset \mathcal{P}(V)$ (the power set of V), such that if $\sigma \in S$ and $\tau \subset \sigma$ then $\tau \in S$. The combinatorial data (V, S) is called an *abstract simplicial complex.* To each such data, there is an associated topological space, called the *geometric realization.* This is constructed inductively on $d \geqslant 0$, by attaching a d-dimensional simplex Δ^d for each element $\sigma \in S$ of cardinality d; see [17]. The result is the simplicial complex K. In practice, we will not distinguish between K and the data (S, V).

Let $K = (V, S)$ be a simplicial complex. Formally, for a subset $S' \subset S$, its *closure* is

$$Cl(S') = \{\tau \in S | \tau \subseteq \sigma \in S'\}$$

The *star* of a simplex $\tau \in S$ is

$$St(\tau) = \{\sigma \in S | \tau \subseteq \sigma\}$$

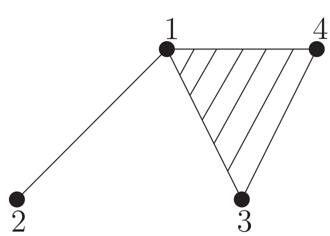
The *link* of a simplex $\tau \in S$ is

$$Lk(\tau) = \{\sigma \in Cl(St(\tau)) | \tau \cap \sigma = \varnothing\}$$

Example 2.1. Let $K = \{V, S\}$, where $V = \{1, 2, 3, 4\}$, and

$$S = \{\{1\}, \{2\}, \{3\}, \{4\}, \{1,2\}, \{1,3\}, \{1,4\}, \{3,4\}, \{1,3,4\}\}.$$

The geometric realization is



The link of $\{4\}$ is the edge $\{1, 3\}$ (including its vertices, of course). The link of $\{1\}$ is the union of $\{2\}$ and the edge $\{3, 4\}$.

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Lectures on monopole Floer homology

Francesco Lin

Abstract. These lecture notes are a friendly introduction to monopole Floer homology. We discuss the relevant differential geometry and Morse theory involved in the definition. After developing the relation with the four-dimensional theory, our attention shifts to gradings and correction terms. Finally, we sketch the analogue in this setup of Manolescu's recent disproof of the long standing Triangulation Conjecture.

Introduction

The present notes are a friendly introduction to monopole Floer homology for low dimensional topologists. The topic has its definitive (and essentially self-contained) reference [11] in which the whole theory is developed in detail. On the other hand, the monograph is quite scary at a first sight, both because of its size and its demanding analytical content (which might be stodgy to many people in the field). Our goal here is to explain the subject without going too deep in the details, and try to convey the key ideas involved. Of course we need to assume some background from the reader. In particular, we expect two things.

- A basic understanding of Seiberg-Witten theory in dimension four, following for example the classic reference [18] (which contains much more than we require). In particular we expect the reader to have digested the differential geometry needed to write down the equations, and to have an idea on how one can use them to define invariants of smooth four manifolds with $b_2^+ \geq 2$.
- A solid understanding of Morse theory in finite dimensions, including the Morse-Witten chain complex. The reader should know how to prove a priori invariance (i.e. without referring to the isomorphism with singular homology) using continuation maps. There are many good references for this, see for example [9] for a nice introduction and [22] for a more thorough discussion.

Roughly speaking, the main complication is that the Seiberg-Witten equations are invariant under an S^1-action which is not free. In usual Morse homology (in finite dimensions), we try to understand the homology of a manifold M using a Morse function f on it. In our case, M comes with an S^1-action and the goal we have in mind is to understand the S^1-equivariant homology of M. To do this, we will introduce a suitable model in Morse homology.

Of course a basic knowledge of the cousin theory Heegaard Floer homology ([21], [20]) will be helpful when dealing with the formal aspects of the theory, but we will not assume that.

The theory has many interesting applications in the study of low dimensional topology. Many of these are already outlined in the last Chapters of [11] and we crafted these notes so that the reader should be able to read those after digesting them. Furthermore the proof of many interesting results in Heegaard Floer homology is formally identical in our setting. For this reason we will build up towards an application which is missing in both setups, namely a disproof of the *Triangulation Conjecture in higher dimensions*. This almost one-hundred-year old problem was settled by Manolescu using his Seiberg-Witten Floer homotopy type approach ([15]). The papers [14] and [16] provide very nice accounts of the background of the problem. In the last few sections of these notes we will build toward the alternative (but formally identical) argument of [13], and we refer the reader to those surveys for a more detailed discussion of the Triangulation Conjecture itself.

Of course there are many sins of omission in the present lectures. Among the others:

- We will not be able to provide interesting examples of computations. Some of these can be obtained using the surgery exact triangle, see [12] and Chapter 42 of [11].
- Throughout the notes, we will forget about orientations of moduli spaces and use only coefficients in $\mathbb{F}$, the field with two elements.
- We will not discuss the applications of this story to the gluing properties of the Seiberg-Witten invariants, which is indeed the original motivation for the definition of the Floer homology groups. This is nicely described in Chapter 3 of [11]. Similarly, the reader can find there a discussion of local coefficients.
- We will not discuss the beautiful non vanishing result which plays a key role in Taubes' proof of the Weinstein conjecture in dimension three ([23]). The details of this are provided in Chapters $33-35$ in [11].

Throughout the lectures we will provide some exercises (with hints) which are worth thinking about. The solution to most of them can be found in [11].

1. The formal picture

We describe the structure of the invariants we will construct. Again we will only consider coefficients in $\mathbb{F}$, the field with two elements. In these notes will focus on closed oriented connected three manifolds. To such a Y we associate three $\mathbb{F}$-vector spaces

$$\widecheck{HM}_*(Y), \quad \widehat{HM}_*(Y), \quad \overline{HM}_*(Y)$$

called the monopole Floer homology groups. These are read respectively *HM-to*, *HM-from* and *HM-bar*. These decompose according to the spin^c structures on the three manifold

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Contact topology from the loose viewpoint

Roger Casals and Emmy Murphy

Abstract. In this expository article, we describe a number of methods for studying high dimensional contact manifolds. We particularly focus on the concept of overtwistedness and looseness and their relation with geometric structures such as open books and surgery cobordisms. These notes are based on a lecture series given by the second author at the 2015 Gökova Geometry-Topology Conference, and built on the series of articles [8, 9, 10].

1. Introduction

1.1. Basics of contact geometry

A *contact structure* on an odd dimensional smooth manifold M^{2n+1} is a maximally non-integrable hyperplane field $\xi^{2n} \subseteq TM$. In this article we will always assume that the hyperplane field ξ is coorientable, i.e. we can write ξ as the kernel of a globally defined 1-form $\alpha \in \Omega^1(M)$. In this case, the condition of maximal non–integrability can be rephrased as the condition that $\alpha \wedge (d\alpha)^n$ is never zero, or equivalently the pair $(\xi, d\alpha)$ is a symplectic bundle. In particular the $(2n+1)$–form $\alpha \wedge (d\alpha)^n$ defines a volume form.

This is an open condition for the hyperplane field ξ and thus a C^1–small perturbation of a contact structure is still a contact structure. In other words, the rank of the 2–form $d\alpha$ has maximal rank when restricted to ξ , which is an open condition in the C^1–topology of the space of hyperplane fields of the tangent bundle TM. This is indeed the generic behaviour of a hyperplane field, in contrast to a situation in which we restrict the rank of $d\alpha$ to be non–maximal. An extreme instance of this latter case is the vanishing $d\alpha|_\xi = 0$, also known as the theory of foliations. See [22] for a discussion in the strongly interesting three–dimensional case.

Remark 1. Though the equality $\xi = \ker \alpha$ will be used systematically, we emphasize that contact structures ξ are a more natural geometric structure in comparison to the 1-form $\alpha \in \Omega^1(M)$ itself, which is only defined by ξ up to multiplication by positive functions (once the coorientation of ξ is fixed). Indeed, the contact condition $\alpha \wedge (d\alpha)^n \neq 0$ is invariant under the transformation $\alpha \rightsquigarrow e^f \alpha$ for any function $f \in C^\infty(M)$, and thus the resulting volume form is an artifact of the particular choice of α. The choice of a particular contact form $\xi = \ker \alpha$ fixes a particular vector field which preserves the contact

Key words and phrases. contact topology, overtwisted contact structures, loose Legendrian.

structures (the Reeb field) and this gives a dynamical flavor to the theory [5].

We do note that the contact structure ξ together with its coorientation determine the sign of the volume form $\alpha \wedge (d\alpha)^n$ and therefore define an orientation on the smooth manifold M. Throughout the article, we will assume that all smooth manifolds M come a priori with an orientation, and by definition a cooriented contact structure is required to induce an orientation which agrees with the prescribed one.

The appearance of contact structures can be traced to the study of geometric optics and wave propagation [2, 6], and classical accounts on the subject can be found in [1, Appendix IV] and [4, Chapter IV]. Since a rotation in an odd dimensional vector space has a fixed axis, contact structures can only exist in odd dimensional smooth manifolds. The counterpart for even dimensional manifolds are symplectic structures, and contact geometry is the odd-dimensional sister of symplectic geometry. In particular, we have the following basic theorems:

Theorem 1.1 ([1]). Let (M, ξ) be a contact manifold, then it is locally equivalent to

$$\mathbb{R}^{2n+1}_{\text{std}} = (\mathbb{R}^{2n+1}, \xi = \ker \alpha_{\text{std}}), \quad \alpha_{\text{std}} = dz - \sum_{i=1}^{n} y_i dx_i.$$

In addition, the moduli of contact structures on a closed manifold is discrete: if ξ_t is a homotopy of contact structures on a closed manifold M, then there is an isotopy $f_t : M \longrightarrow M$ so that $(f_t)_*(\xi_0) = \xi_t$.

The first statement is referred to as Darboux's Theorem, and the second as Gray's stability theorem; these two theorems give contact geometry its marked topological flavor. For instance, Darboux's theorem implies that any contact manifold can alternatively be described by a contact atlas; that is, a smooth atlas where the transition functions are elements of the group of contact transformations

$$\text{Cont}(\mathbb{R}^{2n+1}_{\text{std}}) = \{\varphi \in \text{Diff}(\mathbb{R}^{2n+1}) \mid \varphi^* \alpha_{\text{std}} = e^f \alpha_{\text{std}} \text{ for some } f \in C^\infty M\}.$$

Since the contact condition is C^1–open, Gray's theorem implies that any two contact structures which are sufficiently C^1–close are isotopic, and in particular cut–and–paste operations and corner smoothings are well–defined up to contact isotopy.

1.2. An inadequate history of contact 3–manifolds

There is a long history of connections between 3–dimensional topology and contact structures on 3–manifolds [25]. Martinet established that every 3–manifold admits a contact structure [35], and soon after Lutz [34] showed that in fact a plane field on a 3–manifold is homotopic to a contact structure. In a beautiful application to low–dimensional topology, Eliashberg gave a new proof of the fact that every orientation preserving diffeomorphism of S^3 extends to D^4 using contact geometry [17, Section 6]. These are only three of a fantastic list of results in the establishment of contact geometry as a field [24, 26].

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Noncommutative augmentation categories

Baptiste Chantraine, Georgios Dimitroglou Rizell, Paolo Ghiggini, and Roman Golovko

Abstract. To a differential graded algebra with coefficients in a noncommutative algebra, by dualisation we associate an A_∞-category whose objects are augmentations. This generalises the augmentation category of Bourgeois and Chantraine [2] to the noncommutative world.

1. Introduction

Differential graded algebras (DGAs for short) were introduced by Cartan in [4] and occur naturally in a number of different areas of geometry and topology. We are here interested in those that appear in the context of Legendrian contact homology, which is a powerful contact topological invariant due to Chekanov [6] and Eliashberg, Givental and Hofer [18]. In its basic setup, this theory associates a differential graded algebra, called the *Chekanov-Eliashberg DGA*, to a given Legendrian submanifold of a contact manifold. The DGA homotopy type (or even, stable tame isomorphism type) of the Chekanov-Eliashberg DGA is independent of the choices made in the construction and invariant under isotopy through Legendrian submanifolds. Because of some serious analytical difficulties, Legendrian contact homology has been rigorously defined only for Legendrian submanifolds of contactisations of Liouville manifolds [15] and in few other sporadic cases [6, 21, 32, 26, 17].

Since the Chekanov-Eliashberg DGA is semifree and fully noncommutative, it can be difficult to extract invariants from it. In fact, as an algebra, it is isomorphic to a tensor algebra (and therefore is typically of infinite rank) and its differential is nonlinear with respect to the generators.

To circumvent these difficulties, Chekanov introduced his linearisation procedure in [6]: to a differential graded algebra equipped with an augmentation he associates a chain complex which is generated, *as a module*, by the generators of the DGA *as an algebra*. The differential then becomes linear at the price of losing the information which is contained in the multiplicative structure of the DGA, but at least the homology of the linearised

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complex is computable. It is well known that the set of isomorphism classes of linearised homologies is invariant under DGA homotopy; see e.g. [1, Theorem 2.8]. Thus, linearised Legendrian contact homology provides us with a computable Legendrian isotopy invariant.

In order to recover at least part of the nonlinear information lost in the linearisation, one can study products in the linearised Legendrian contact homology groups induced by the product structure of the Chekanov-Eliashberg DGA.

Civan, Koprowski, Etnyre, Sabloff and Walker in [8] endowed Chekanov's linearised chain complex with an A_∞-structure. This construction was generalised in [2] by the first author and Bourgeois, who showed that a differential graded algebra naturally produces an A_∞-category whose objects are its augmentations. In dimension three, the A_∞-category constructed by the first author and Bourgeois admits a unital refinement defined by Ng, Rutherford, Shende, Sivek and Zaslow in [29]. The latter article also establishes an equivalence between this unital A_∞-category and one defined in terms of derived sheaves of microlocal rank one with microsupport given by a fixed Legendrian knot. Our expectation is that the A_∞-structures constructed here correspond to such sheaves being of arbitrary microlocal rank.

A_∞-algebras are by now classical structures which were first introduced by Stasheff in [36] as a tool in the study of 'group-like' topological spaces. Fukaya was the first to upgrade the notion of an A_∞-algebra to that of an A_∞-category. In [23] he associated an A_∞-category, which now goes under the name of the *Fukaya category*, to a symplectic manifold. See [33] for a good introduction. Inspired by Fukaya's work [23], Kontsevich in [25] formulated the *homological mirror symmetry conjecture* relating the derived Fukaya category of a symplectic manifold to the derived category of coherent sheaves on a "mirror" manifold.

The construction in [8] and [2] defines A_∞-operations only when the coefficient ring of the DGA is commutative. The goal of this paper is to extend that construction to noncommutative coefficient rings in the following two cases:

(I) the coefficients of the DGA as well as the augmentations are taken in a unital noncommutative algebra, or

(II) the coefficients of the DGA as well as the augmentations are taken in a noncommutative *Hermitian algebra*. (See Definition 2.1.) This case includes both finite-dimensional algebras over a field and group rings.

Case (II) is obviously included in Case (I), but we will see that there is a particularly nice alternative construction of an A_∞-structure in case (II) which gives a different result. We refer to Subsections 4.1 and 4.2 for the respective constructions. Both generalisations above are sensible to study when having Legendrian isotopy invariants in mind, albeit for different reasons.

Case (I) occurs because there are Legendrian submanifolds whose Chekanov-Eliashberg DGA does not admit augmentations in any unital algebra of finite rank over a commutative ring, but admits an augmentation in a unital noncommutative infinite-dimensional one (for example, in their characteristic algebras). The first such examples were Legendrian knots constructed by Sivek in [35] building on examples found by Shonkwiler and

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Uniqueness of extremal Lagrangian tori in the four-dimensional disc

Georgios Dimitroglou Rizell

Abstract. The following interesting quantity was introduced by K. Cieliebak and K. Mohnke for a Lagrangian submanifold L of a symplectic manifold: the minimal positive symplectic area of a disc with boundary on L. They also showed that this quantity is bounded from above by π/n for a Lagrangian torus inside the $2n$-dimensional unit disc equipped with the standard symplectic form. A Lagrangian torus for which this upper bound is attained is called extremal. We show that all extremal Lagrangian tori inside the four-dimensional unit disc are contained in the boundary $\partial D^4 = S^3$. It also follows that all such tori are Hamiltonian isotopic to the product torus $S^1_{1/\sqrt{2}} \times S^1_{1/\sqrt{2}} \subset S^3$. This provides an answer to a question by L. Lazzarini in the four-dimensional case.

1. Introduction and results

In the following we will consider the standard even dimensional symplectic vector space $(\mathbb{C}^n, \omega_0 := dx_1 \wedge dy_1 + \ldots + dx_n \wedge dy_n)$, as well as the projective space $(\mathbb{C}P^n, \omega_{\mathrm{FS},r})$ endowed with the Fubini-Study symplectic two-form. We here normalise $\omega_{\mathrm{FS},r}$ so that a line $\ell \subset \mathbb{C}P^n$ has symplectic area equal to $\int_\ell \omega_{\mathrm{FS},r} = \pi r^2$. We also write $\omega_{\mathrm{FS}} := \omega_{\mathrm{FS},1/\sqrt{\pi}}$. See Section 2 for more details.

Neck-stretching techniques were successfully used in [6] by K. Cieliebak and K. Mohnke in order to prove the Audin conjecture, first formulated in [1] by M. Audin: Every Lagrangian torus in $\mathbb{C}^n$ or $\mathbb{C}P^n$ bounds a disc of positive symplectic area and Maslov index equal to two. The same techniques were also used to deduce properties concerning the following quantity for a Lagrangian submanifold, which was introduced in the same article. (We here restrict our attention to Lagrangian tori.) Given a Lagrangian torus $L \subset (X, \omega)$ inside an arbitrary symplectic manifold, we define

$$A_{\min}(L) := \inf_{\substack{A \in \pi_2(X,L) \\ \int_A \omega > 0}} \int_A \omega \in [0, +\infty].$$

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Key words and phrases. Capacities, Extremal Lagrangian tori, monotone Lagrangian tori.

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This quantity can then be used in order to define a capacity for the symplectic manifold (X, ω) as follows:

$$c_{\mathrm{Lag}}(X,\omega) := \sup_{L\subset(X,\omega)\ \text{Lag. torus}} A_{\min}(L) \in [0,+\infty].$$

We refer to [6] for the properties satisfied by this capacity. In view of this it is natural to consider:

Definition 1.1 ([6])**.** A Lagrangian torus $L \subset (X, \omega)$ satisfying

$$A_{\min}(L) = c_{\mathrm{Lag}}(X,\omega)$$

is called *extremal*.

The above capacity has been computed only for a limited number of symplectic manifolds, notably:

Theorem 1.1 (Theorem 1.1 and Corollary 1.3 in [6])**.** *We have*

$$c_{\mathrm{Lag}}(B^{2n},\omega_0) = \pi/n, \tag{1}$$
$$c_{\mathrm{Lag}}(\mathbb{C}P^n,\omega_{\mathrm{FS},r}) = r^2\pi/(n+1), \tag{2}$$

and in particular $c_{\mathrm{Lag}}(D^{2n},\omega_0) = \pi/n$.

A straight-forward calculation shows that the n-dimensional Clifford torus

$$L_{\mathrm{Cl}} := \left(S^1_{\frac{1}{\sqrt{n}}}\right)^n \subset S^{2n-1} = \partial D^{2n} \subset (\mathbb{C}^n,\omega_0),$$

contained inside the boundary of the $2n$-dimensional unit disc is extremal. In the case when $n = 1$, the Clifford torus is clearly the only extremal Lagrangian torus. Furthermore, a monotone Lagrangian torus $L \subset (\mathbb{C}P^n, \omega_{\mathrm{FS}})$ is extremal, as follows by elementary topological considerations together with the fact that there exists a representative of $\pi_2(\mathbb{C}P^2, L)$ having Maslov index two and positive symplectic area by [6, Theorems 1.1, 1.2]. (For previous related results, consider [23], [19], [18], [12], [4], and [8].)

In [6] the author learned about the following two conjectures, the first one originally due to L. Lazzarini:

Conjecture 1.2. All extremal Lagrangian tori $L \subset (D^{2n}, \omega_0)$ are contained inside the boundary $\partial D^{2n} = S^{2n-1}$.

Conjecture 1.3. All extremal Lagrangian tori $L \subset (\mathbb{C}P^n, \omega_{\mathrm{FS}})$ are monotone.

Our main result is a positive answer to Conjecture 1.2 in dimension four.

Theorem 1.2. *All extremal Lagrangian tori* $L \subset (D^4, \omega_0)$ *are contained inside the boundary, i.e.* $L \subset S^3 = \partial D^4$.

After a consideration of the possible Lagrangian tori inside the three-dimensional unit sphere using classical techniques, we also obtain the following classification result.

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An introduction to tangle Floer homology

Ina Petkova and Vera Vértesi

Abstract. This paper is a short introduction to the combinatorial version of tangle Floer homology defined in [PV14]. There are two equivalent definitions—one in terms of strand diagrams, and one in terms of bordered grid diagrams. We present both, discuss the correspondence, and carry out some explicit computations.

1. Introduction

Knot Floer homology is a categorification of the Alexander polynomial. It was introduced by Ozsváth–Szabó [OS04] and Rasmussen [Ras03] in the early 2000s. One associates a bigraded chain complex $\widehat{\mathrm{CFK}}(\mathcal{H})$ over $\mathbb{F}_2$ to a Heegaard diagram $\mathcal{H} = (\Sigma, \boldsymbol{\alpha}, \boldsymbol{\beta}, \mathbf{z}, \mathbf{w})$ for a link L. The generators are combinatorial and can be read off from the intersections of curves on the Heegaard diagram, whereas the differential counts pseudoholomorphic curves in $\Sigma \times I \times \mathbb{R}$ satisfying certain boundary conditions. The homology of $\widehat{\mathrm{CFK}}(\mathcal{H})$ is an invariant of L denoted $\widehat{\mathrm{HFK}}(L)$.

Knot Floer homology is a powerful link invariant—it detects genus, detects fiberedness, and an enhanced version called HFK^- contains a concordance invariant $\tau(K) \in \mathbb{Z}$, whose absolute value bounds the 4-ball genus of K, and hence the unknotting number of K.

Combinatorial versions of knot Floer homology [MOS09, MOST07] were defined soon after the original construction, but they were still global in nature, and our understanding of how local modifications of a knot affect HFK was very limited [Man07, OS07].

In [PV14], we "localize" the construction of knot Floer homology, and define an invariant of oriented tangles. Although we develop a theory for oriented tangles in general 3-manifolds with spherical boundaries by using analysis similar to [LOT08, LOT10], in this paper we will focus on a completely combinatorial construction for tangles in B^3 and $I \times S^2$ (we'll think of those as tangles in $I \times \mathbb{R}^2$).

An (m,n)-*tangle* $\mathcal{T}$ is a proper, smoothly embedded oriented 1–manifold in $I \times \mathbb{R}^2$, with boundary $\partial\mathcal{T} = \partial^L\mathcal{T} \sqcup \partial^R\mathcal{T}$, where $\partial^L\mathcal{T} = \{0\} \times \{1, \dots, m\} \times \{0\}$ and $\partial^R\mathcal{T} = \{1\} \times \{1, \dots, n\} \times \{0\}$, treated as oriented sequences of points; if m or n is zero, the respective boundary is the empty set. A planar diagram of a tangle is a projection to $I \times \mathbb{R} \times \{0\}$ with no triple intersections, self-tangencies, or cusps, and with over- and under-crossing data preserved (as viewed from the positive z direction). The boundaries

Key words and phrases. tangles, knot Floer homology, TQFT.

of $\mathcal{T}$ can be thought of as *sign sequences*

$$-\partial^L\mathcal{T} \in \{+,-\}^m, \quad \partial^R\mathcal{T} \in \{+,-\}^n,$$

according to the orientation of the tangle at each point (+ if the tangle is oriented left-to-right, − if the tangle is oriented right-to-left at that point). See for example Figure 1. Given two tangles $\mathcal{T}$ and $\mathcal{T}'$ with $\partial^R\mathcal{T} = -\partial^L\mathcal{T}'$, we can concatenate them to obtain a

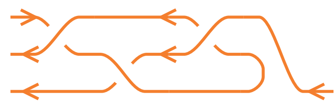

FIGURE 1. A projection of a (3,1)-tangle $\mathcal{T}$ to $I \times \mathbb{R}$. Here $-\partial^L\mathcal{T} = (-,-,+)$ and $\partial^R\mathcal{T} = (-)$.

new tangle $\mathcal{T} \circ \mathcal{T}'$, by placing $\mathcal{T}'$ to the right of $\mathcal{T}$.

We associate a differential graded algebra called $\mathcal{A}^-(P)$ to a sign sequence $P \in \{+,-\}^n$, and a type *DA* bimodule $\mathrm{CT}^-(\mathbb{T})$ over $(\mathcal{A}^-(-\partial^L\mathcal{T}), \mathcal{A}^-(\partial^R\mathcal{T}))$ to a fixed *decomposition* $\mathbb{T}$ of an (m,n)-tangle $\mathcal{T}$. These structures come equipped with a grading M by $\mathbb{Z}$, called the Maslov grading, and a grading A by $\frac{1}{2}\mathbb{Z}$, called the Alexander grading. Setting certain variables U_i in CT^- to zero, we get a simpler bimodule $\widetilde{\mathrm{CT}}$, which we prove in [PV14] to be an invariant of the tangle $\mathcal{T}$ (there is evidence suggesting that CT^- is an invariant too, but we do not at present have a complete proof). Gluing corresponds to taking box tensor product, and for closed links the invariant recovers HFK:

Theorem 1.1. *Given an (l,m)-tangle $\mathcal{T}_1$ with decomposition $\mathbb{T}_1$ and an (m,n)-tangle $\mathcal{T}_2$ with decomposition $\mathbb{T}_2$ with $\partial^R\mathcal{T}_1 = -\partial^L\mathcal{T}_2$, let $\mathbb{T} = \mathbb{T}_1\mathbb{T}_2$ be the corresponding decomposition for the concatenation $\mathcal{T}_1 \circ \mathcal{T}_2$. Then there is a bigraded isomorphism*

$$\mathrm{CT}^-(\mathbb{T}_1) \boxtimes_{\mathcal{A}^-(\partial^R\mathcal{T}_1)} \mathrm{CT}^-(\mathbb{T}_2) \simeq \mathrm{CT}^-(\mathbb{T}).$$

Regarding an l-component link $\mathcal{L}$ (with some decomposition $\mathbb{L}$) as a $(0,0)$-tangle, we have

$$\mathrm{CT}^-(\mathbb{L})[-l/2]\{-l/2\} \simeq gCFK^-(\mathcal{L}) \otimes (\mathbb{F}_2[-1/2] \oplus \mathbb{F}_2[1/2]),$$

where $[i]$ denotes a Maslov grading shift down by i, and $\{j\}$ an Alexander grading shift down by j.

We define CT^- combinatorially, by means of *bordered grid diagrams*, or, equivalently, *strand diagrams*.

1.1. Outline

In Section 2 we describe the two constructions for CT^- and discuss their correspondence. Section 3 contains a couple of small explicit computations (a cap and a cup, which glue up to the unknot).

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Thoughts about a good classification of manifolds

Matthias Kreck

Abstract. "Good" is a matter of taste. But there are mathematical concepts which most mathematicians agree on that they are good. For example if one wants to classify complex vector bundles it seems to be a good idea to look at isomorphism classes of vector bundles modulo addition of trivial vector bundles. The resulting set of equivalence classes is denoted by $\tilde{K}^0(X)$, and for compact spaces this is a group under the operation of Whitney sum. Another reason why it is good, is that $\tilde{K}^0(X)$ is the degree 0 subgroup of a generalized cohomology theory which allows an attack by the standard tools of algebraic topology like exact sequences or spectral sequences.

In this note we take this as a model for the classification of closed connected manifolds. In analogy to the vector bundles we consider diffemorphism classes of smooth manifolds modulo connected sum with a "trivial" manifold T. Whereas we don't see a good candidate for T for odd dimensional manifolds, we take $T = S^n \times S^n$ in even dimensions and pass to what we call the reduced stable diffeomorphism classes of manifolds. In contrast to vector bundles the reduced stable diffeomorphism classes of smooth manifolds don't form a group. But we will see that they decompose as quotients of groups by a linear action of another group. Most of the results in this note are not new, they are all based on the results of my papers [13], [14]. But we add a perspective which readers might find good.

1. The model K-theory

Let X be a compact topological space. We denote the set of isomorphism classes of finite dimensional complex vector bundles over X by $Vect(X)$. This is a monoid under the Whitney sum. Computations of $Vect(X)$ are at present not accessible, even in the case of spheres, where it is equivalent to the computation of (unstable) homotopy groups of the orthogonal groups $O(n)$. But if $k > n$, the set of isomorphism classes of k-dimensional vector bundles over S^n can be computed. The set is independent of $k > n$ and was computed by Bott with his periodicity theorem: It is trivial if n is odd, and $\mathbb{Z}$ if n is even.

More generally, if one considers for a compact space X the equivalence relation **reduced stable equivalence**, which means that two complex vector bundles E and E' are reduced stably equivalent, if there are integers k and l, such that $E \oplus \mathbb{C}^k \cong E' \oplus \mathbb{C}^l$, the reduced stable equivalence classes form a group denoted by $\tilde{K}^0(X)$. The point here is that all finite dimensional vector bundles can be embedded into a trivial vector bundle, which implies that if the addition on the reduced stable equivalence classes is induced by

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the Whitney sum, all vector bundles have an inverse in $\tilde{K}^0(X)$. Using Bott periodicity Atiyah and Hirzebruch [2], [1] extend this to a reduced generalized cohomology theory $\tilde{K}^*(X)$.

It would be nice to define a similar relation on the set of diffeomorphism classes of closed smooth connected manifolds of a fixed dimension n, which makes the set of equivalence classes a group with a geometrically defined group structure. Let's look at the case of surfaces. There the obvious candidate for an addition is the connected sum which should play the role of the Whitney sum for vector bundles. The next step would be to find a replacement for the trivial bundle, a manifold T: the "trivial manifold" replacing the trivial vector bundle. Choosing such a T one can call surfaces F and F' **reduced T-stably isomorphic** if there are integers k and l such that $F\sharp kT$ is diffeomorphic to $F'\sharp lT$. Now one can look for a T, such that reduced T-stable isomorphism classes form a group. We would need a neutral element N which means that $F\sharp N\sharp kT$ is diffeomorphic to $F\sharp lT$ for some k and l, and an inverse, which means that for each F there is an F' such that $F\sharp F'\sharp kT$ is diffeomorphic to $N\sharp lT$ for some k and l. If we could find such a T which is orientable, then also N must be orientable and then a non-orientable manifold has no inverse. Thus one has to take T non-orientable, for example $T = \mathbb{R}P^2$. Then it works, but the result is disappointing since the resulting group is the trivial group.

But this is the only dimension, where this can work. In dimension $n \geq 3$ such a T cannot exist. If so, there would be a manifold N representing the neutral element. If we would have a group structure on the reduced stable isomorphism classes for each closed connected n-manifold M there is a closed n-manifold M' - the inverse - such that $M\sharp M'\sharp rT \cong N\sharp lT$ for some k and l, which alone from the point of view of homology groups is impossible. Namely, *tors* $H_1(T)$ and *tors* $H_1(N)$ are finite and so there is a prime p, such that $H_1(T)$ and $H_1(N)$ have no p-torsion, and so a manifold M with non-trivial p-torsion has no inverse.

2. A weakening of the model of vector bundles

In odd dimensions I don't have any good idea what to do. But the following result by Wall about 1-connected oriented 4-manifolds is a guide to a modification. In his case $T = S^2 \times S^2$. Instead of "reduced $S^2 \times S^2$-stably equivalent" we just write "reduced stably diffeomorphic".

Theorem 2.1. [22] *Two closed smooth* 1*-connected oriented* 4*-manifolds M and M' are reduced stably diffeomorphic if and only*
- sign(M) = sign(M')
- both are Spin or both do not admit a Spin-structure.

This suggests to consider two disjoint subsets of the reduced stable diffeomorphism classes of closed simply connected 4-manifolds: $\mathcal{M}(1)$, the reduced stable diffeomorphism classes of 1-connected Spin manifolds and $\mathcal{M}(2)$, the reduced stable diffeomorphism classes of 1-connected manifolds admitting no Spin structure. Then Wall's theorem implies that both sets are actually groups under the operation given by connected sum, and both are

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Nonexistence of rational homology disk weak fillings of certain singularity links

Mohan Bhupal and András I. Stipsicz

Abstract. We show that the Milnor fillable contact structures on the links of singularities having resolution graphs from some specific families that have members with arbitrarily large numbers of nodes do not admit weak symplectic fillings having the rational homology of the 4-disk. This result provides further evidence toward the conjecture that no such weak symplectic filling exists once the mininal resolution tree has at least two nodes.

1. Introduction

Isolated complex surface singularities and their smoothings (if they exist) play an important role in constructing interesting smooth 4-manifolds. In particular, suppose that $(S, 0)$ is an isolated surface singularity with resolution graph Γ and M is the Milnor fibre of a smoothing of S. Suppose, furthermore, that (X, ω) is a symplectic 4-manifold and $C_1, \ldots, C_n \subset (X, \omega)$ are symplectic submanifolds intersecting each other according to Γ (and ω-orthogonally). It was shown in [13] that

$$Z = (X - \nu(C_1 \cup \ldots \cup C_n)) \cup M$$

is also symplectic. In many cases the underlying smooth structure is *exotic*, that is, homeomorphic but not diffeomorphic to some standard smooth 4-manifold. This construction was discovered and exploited by Fintushel and Stern [6] (and extended by J. Park [15]) for cyclic quotient singularities and Milnor fibres admitting the rational homology of a disk — this is Fintushel and Stern's *rational blow-down* procedure.

A smoothing with Milnor fibre M satisfying $H_*(M; \mathbb{Q}) \cong H_*(D^4; \mathbb{Q})$ is called a *rational homology disk smoothing* (or $\mathbb{Q}$HD smoothing for short). The list of singularities with $\mathbb{Q}$HD smoothings was conjectured by Wahl (cf. [8]); this conjecture was verified in [2] for weighted homogeneous singularities, that is, singularities which admit resolution graphs with one node. (Recall that a *node* in a graph in this context means a vertex with valency at least three.) Using different methods, this result was extended by Wahl [18] to singularities with resolution graphs having two nodes, each with valency 3. In [14] a complete solution of the problem is claimed: it is shown that a singularity with minimal resolution graph admitting at least two nodes does not have a $\mathbb{Q}$HD smoothing, hence the result of [2] implies the resolution of Wahl's conjecture.

In fact, the result verified in [2] is slightly stronger: the argument provides a classification of resolution graphs with one node which are rational and for which the corresponding Milnor fillable contact structure on the link admits a weak symplectic $\mathbb{Q}$HD filling. (The methods of [18] and [14], being algebro-geometric in nature, do not provide any information about weak symplectic fillings.) For completeness, let us recall the definition of weak and strong symplectic fillings (cf. also [7, Definition 5.1.1] or [12, Definition 12.1.1]).

Definition 1.1. For a given contact 3-manifold (Y,ξ) (where ξ is assumed to be cooriented), the compact symplectic 4-manifold (X,ω) is a **weak symplectic filling** if $\partial X = Y$ as oriented manifolds (where X is oriented by the volume form $\omega\wedge\omega$), and $\omega|_\xi > 0$. The compact symplectic 4-manifold (X,ω) is a **strong symplectic filling** if $\partial X = Y$ and there is a Liouville vector field V on X near ∂X satisfying $\xi = \ker(\iota_V\omega|_{TY})$. Here $\iota_V\omega$ denotes the contraction of the 2-form ω with the vector field V and the vector field being Liouville means that the Lie derivative $\mathfrak{L}_V\omega$ of the 2-form ω along V is ω.

Remark 1.2. Obviously a strong filling is a weak filling; moreover, for a strong filling the symplectic structure ω of the filling near the boundary ∂X is exact. According to [4, Proposition 3.1] (see also [5, Lemma 2.1] or [10]) this is essentially the only obstruction for a weak filling to be strong: if ω is exact near ∂X then (according to [4]) we can attach a topologically trivial symplectic cobordism $(\widetilde{X},\tilde{\omega})$ to $(\partial X,\omega)$ which is now a strong symplectic filling of (Y,ξ). Notice that if $b_1(Y)=0$ (as will be always the case for the 3-manifolds considered in the following), then the exactness hypothesis near ∂X is automatically satisfied.

The argument presented in [17] shows that if Γ is the resolution graph of a singularity with Milnor fillable contact structure admitting a weak symplectic $\mathbb{Q}$HD filling and Γ admits more than one node, then Γ is a member of one of the inductively defined families $\mathcal{A}$, $\mathcal{B}$ or $\mathcal{C}$ of graphs described in [17]. (Here we will not repeat the simple definition of the graph families, see [17] or [14].)

In the present work we describe three families of resolution graphs Γ_n^C, Γ_n^B and Γ_n^A (one from each of the families $\mathcal{C}$, $\mathcal{B}$ and $\mathcal{A}$ of [17])), such that Γ_n^* (with $*=C,B,A$) has $n+2$ nodes and, although infinitely many of them pass the combinatorial constraints of [16, 17], the corresponding Milnor fillable contact structures on the plumbing 3-manifolds admit no weak symplectic $\mathbb{Q}$HD fillings (with the additional assumption $n\neq 0,3,5$ or 6 for the family Γ_n^A). The proof slightly extends the idea of the proof of the main result of [2], and relies on the same fundamental theorem of McDuff concerning symplectic 4-manifolds containing symplectic spheres of self-intersection $+1$.

The novelty of the present work is that in [2] we managed to find a concave filling of the Milnor fillable contact 3-manifold at hand that was diffeomorphic to the tubular neighbourhood of a configuration of J-holomorphic curves (for some almost complex

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Non semi-simple TQFTs from unrolled quantum $sl(2)$

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Abstract. Invariants of 3-manifolds from a non semi-simple category of modules over a version of quantum $\mathfrak{sl}(2)$ were obtained by the last three authors in [4]. They are invariants of 3-manifolds together with a cohomology class which can be interpreted as a line bundle with flat connection. In [1] we have extended those invariants to graded TQFTs on suitable cobordism categories. Here we give an overview of constructions and results, and describe the TQFT vector spaces. Then we provide a new, algebraic, approach to the computation of these vector spaces.

Introduction

New quantum invariants of 3-manifolds equipped with 1-dimensional cohomology classes over $\mathbb{C}/2\mathbb{Z}$, or equivalently $\mathbb{C}^*$ flat connections, have been constructed in [4] from a variant of quantum $\mathfrak{sl}(2)$. This family of invariants is indexed by integers $r \geq 2$, $r \not\equiv 0$ mod 4, which gives the order of the quantum parameter. In the case $r \equiv 0$ mod 4, we have obtained in [2] invariants of 3-manifolds equipped with generalised spin structures corresponding to certain flat connections on the oriented framed bundle. These invariants are built from surgery presentations and have common flavor with the famous Witten-Reshetikhin-Turaev quantum invariants, but are indeed very different. First, they are defined for 3-manifolds equipped with cohomology classes, and second they use a stronger version of quantum $sl(2)$ which in particular avoids the semi-simplification procedure required for producing modular categories. To emphasize the power of these new invariants we quote that for the smallest root of unity, $r = 2$, the multivariable Alexander polynomial and Reidemeister torsion are recovered, which allows us to reproduce the classification of lens spaces, see [1].

The TQFT extension of these invariants has been carried out in [1]. The main achievement is a functor on a category of *decorated cobordisms* with values in finite dimensional graded vector spaces. An object in this category is a surface equipped with the following data: a base point on each connected component, a possibly empty set of colored points, a 1-dimensional cohomology class over $\mathbb{C}/2\mathbb{Z}$ and a Lagrangian subspace. A morphism is a cobordism with: a colored ribbon graph, a cohomology class and a signature defect which all satisfy certain admissibility conditions. A description of these TQFT vector spaces split into two cases, depending on whether the cohomology class is integral or not. In the non-integral case, it can be done using colored trivalent graphs with a pattern similar to the Witten-Reshetikhin-Turaev case. In the integral case we are able to prove finite dimensionality in general and to provide a Verlinde formula for their graded dimension under the further assumption that the surface contains a point with a projective color. A new result in this paper is an Hochschild homology description of the TQFT vector spaces. In the integral case, our statement is proved under the previous assumption.

1. Unrolled quantum $sl(2)$ and modified trace invariant

In this section we recall the unrolled quantum $sl(2)$ at a root of unity. In the whole paper, $r \geq 2$ is an integer which is non zero modulo 4, $r' = r$ if r is odd and $r' = \frac{r}{2}$ else.

Let $q = \mathrm{e}^{\frac{i\pi}{r}}$, $q^x = \mathrm{e}^{\frac{ix\pi}{r}}$ for $x \in \mathbb{C}$. Recall (see [5]) the $\mathbb{C}$-algebra $\overline{U}_q^H\mathfrak{sl}(2)$ given by generators E, F, K, K^{-1}, H and relations:

$$KEK^{-1} = q^2E, \qquad KFK^{-1} = q^{-2}F, \qquad [E,F] = \frac{K-K^{-1}}{q-q^{-1}}, \qquad E^r = 0,$$
$$HK = KH, \qquad [H,E] = 2E, \qquad [H,F] = -2F, \qquad F^r = 0.$$

The algebra $\overline{U}_q^H\mathfrak{sl}(2)$ is a Hopf algebra where the coproduct, counit and antipode are defined in [5]. A *weight module* is a finite dimensional module which splits as a direct sum of H-weight spaces and is such that K acts as q^H.

The category $\mathscr{C}$ of weight modules is $\mathbb{C}/2\mathbb{Z}$-graded (by the weights modulo $2\mathbb{Z}$) that is $\mathscr{C} = \bigoplus_{\overline{\alpha}\in\mathbb{C}/2\mathbb{Z}} \mathscr{C}_{\overline{\alpha}}$ and $\otimes : \mathscr{C}_{\overline{\alpha}} \times \mathscr{C}_{\overline{\beta}} \to \mathscr{C}_{\overline{\alpha}+\overline{\beta}}$. The category $\mathscr{C}$ is a ribbon category and we have the usual Reshetikhin-Turaev functor from $\mathscr{C}$-colored ribbon graphs to $\mathscr{C}$ (which is given by Penrose graphical calculus).

The simple modules in $\mathscr{C}$ are highest weight modules with any complex number as highest weight. The generic simple modules are those which are projective. They are indexed by the set

$$\ddot{\mathbb{C}} = (\mathbb{C} \setminus \mathbb{Z}) \cup r\mathbb{Z}.$$

For $\alpha \in \ddot{\mathbb{C}}$, the r-dimensional module $V_\alpha \in \mathscr{C}_{\overline{\alpha+r-1}}$ is the irreducible module with highest weight $\alpha + r - 1$.

The group of invertible modules is generated by the one dimensional vector space denoted by $\varepsilon = \mathbb{C}_r$ with H-weight equal to r and degree equal to r modulo 2. The subgroup of invertible objects with trivial degree is generated by $\sigma = \mathbb{C}_{2r'}$, the one dimensional vector space with H-weight equal to $2r'$ (if r is even then $\sigma = \varepsilon$). For each integer j, $0 \leq j \leq r-1$ the simple module with highest weight j is $j+1$ dimensional. For $0 \leq j < r-1$ it is not projective, but has a $2r$-dimensional projective cover P_j. The non simple indecomposable projective modules are the $P_j \otimes \mathbb{C}_r^{\otimes k}$, $0 \leq j < r-1$, $k \in \mathbb{Z}$.

The link invariant underlying our construction is the re-normalized link invariant ([8]) that we recall briefly. The modified dimension is the function defined on $\{V_\alpha\}_{\alpha\in\ddot{\mathbb{C}}}$ by

$$\mathsf{d}(\alpha) = (-1)^{r-1}\frac{r\{\alpha\}}{\{r\alpha\}},$$

where $\{\alpha\} = 2i\sin\frac{\pi\alpha}{r}$. Let L be a $\mathscr{C}$-colored oriented framed link in S^3 with at least one component colored by an element of $\{V_\alpha : \alpha \in \ddot{\mathbb{C}}\}$. Opening such a component of L gives

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