

Selected Works of
Isadore Singer

VOLUME 1

FUNCTIONAL ANALYSIS, DIFFERENTIAL GEOMETRY
AND EIGENVALUES

edited by

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Selected Works of Isadore Singer
Volume 1: Functional Analysis, Differential Geometry and Eigenvalues

One of a set of three volumes.

Editors:

Daniel S. Freed (University of Texas, Austin)

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Preface to the Selected Works

In these three volumes we present the major mathematical work of Isadore Singer. We have divided the papers into three thematic volumes, which largely follow chronological order. These pages convey Is' adventurous spirit, constant innovation, and deep mathematical insight. Is made the selection of papers to include in this collection. We have added brief commentaries as a guide to the reader.

It is a pleasure to thank S. T. Yau for encouraging us to put this collection together and for his support and advice throughout. Rob Kirby and *Celebratio Mathematica* kindly agreed to let us include Rob's 2019 biography of Is, a vivid portrait which brings to life the man behind the work. We also thank Sorin Popa and Dan Voiculescu for their mathematical advice. Finally, we express our gratitude to Is' family, especially Rosemarie, Eliot, Natasha, Emily, Annabelle, Giles and Melissa, for their support. We dedicate these volumes to them and to the memory of Is.

Daniel S. Freed and John Lott
July 2021

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