

*Selected Works of*  
Isadore Singer

VOLUME 3  
GAUGE THEORY AND PHYSICS

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*edited by*

Daniel S. Freed  
University of Texas, Austin

John Lott  
University of California, Berkeley

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Selected Works of Isadore Singer  
Volume 3: Gauge Theory and Physics

One of a set of three volumes.

Editors:

Daniel S. Freed (University of Texas, Austin)

John Lott (University of California, Berkeley)

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## Preface to the Selected Works

In these three volumes we present the major mathematical work of Isadore Singer. We have divided the papers into three thematic volumes, which largely follow chronological order. These pages convey Is' adventurous spirit, constant innovation, and deep mathematical insight. Is made the selection of papers to include in this collection. We have added brief commentaries as a guide to the reader.

It is a pleasure to thank S. T. Yau for encouraging us to put this collection together and for his support and advice throughout. Rob Kirby and *Celebratio Mathematica* kindly agreed to let us include Rob's 2019 biography of Is, a vivid portrait which brings to life the man behind the work. We also thank Sorin Popa and Dan Voiculescu for their mathematical advice. Finally, we express our gratitude to Is' family, especially Rosemarie, Eliot, Natasha, Emily, Annabelle, Giles and Melissa, for their support. We dedicate these volumes to them and to the memory of Is.

Daniel S. Freed and John Lott  
July 2021



# Contents of the Current Volume

## VOLUME 3 GAUGE THEORY AND PHYSICS

PREFACE TO THE SELECTED WORKS . . . . .	v
CURRICULUM VITAE . . . . .	xiii
LIST OF PUBLICATIONS . . . . .	xv
SINGER'S WORK ON GEOMETRY AND QUANTUM FIELD THEORY . . . . .	xxv

### PAPERS

36. with M. F. Atiyah, and N. J. Hitchin, "Self-duality in four-dimensional Riemannian geometry," in <i>Proc. Roy. Soc. London, Series A</i> <b>362</b> :1711 (1978), 425–461. MR 506229 Zbl 0389.53011 . . . . .	<b>763</b>
37. "Some remarks on the Gribov ambiguity," <i>Comm. Math. Phys.</i> <b>60</b> (1978), 7–12. MR 500248 Zbl 0379.53009 . . . . .	<b>801</b>
38. "Differential geometry, fiber bundles, and physical theories," <i>Physics Today</i> <b>35</b> (1982), 41–44. . . . .	<b>807</b>
39. with Orlando Alvarez and Bruno Zumino, "Gravitational anomalies and the family's index theorem," <i>Comm. Math. Phys.</i> <b>96</b> :3 (1984), 409–417. MR 769356 Zbl 0587.58042 . . . . .	<b>811</b>
40. with M. F. Atiyah, "Dirac operators coupled to vector potentials," in <i>Proc. Nat. Acad. Sci. U.S.A.</i> <b>81</b> :8 (1984), Phys. Sci., 2597–2600. MR 742394 Zbl 0547.58033. . . . .	<b>821</b>
41. "Families of Dirac operators with applications to physics," in <i>The Mathematical Heritage of Élie Cartan</i> (Numero Hors Série, Asterisque, Lyon, 1985), 323–340. MR 837207 Zbl 0603.58054. . . . .	<b>825</b>
42. "The $\eta$ -invariant and the index," in <i>Adv. Ser. Math. Phys.</i> <b>1</b> (World Sci. Publishing, Singapore, 1987), 239–258. MR 915824 Zbl 0669.58036. . . . .	<b>843</b>
43. With L. Baulieu, "The topological sigma model," <i>Comm. Math. Phys.</i> <b>125</b> :2 (1989), 227–237. MR 1016870 Zbl 0684.57016 . . . . .	<b>863</b>

44. “The new mathematical physics,” in *Elementary Particles and the Universe: Essays in Honor of Murray Gell-Mann* (Cambridge Univ. Press, 1991), 187–191. MR 1145030 . . . . . **875**
45. with Scott Axelrod, “Chern-Simons perturbation theory,” talk given at the XXth International Conference on Differential Geometric Methods in Theoretical Physics, New York, June 1991. Preprint. arXiv:hep-th/9110056. MR 1225107 Zbl 0813.53051 . . . . . **881**
46. with Scott Axelrod, “Chern-Simons perturbation theory II,” *J. Diff. Geom.* **39:1** (1994), 173–213. MR 1258919 Zbl 0827.53057 . . . . . **917**
47. with M. J. Hopkins, “Quadratic functions in geometry, topology, and  $M$ -theory,” *J. Diff. Geom.* **70:3** (2005) 329–452. MR 2192936 Zbl 1116.58018 . . . . . **959**

**APPENDIX**

- ACKNOWLEDGEMENTS . . . . . **A-1**

# Contents of Volumes 1 and 2

## VOLUME 1 FUNCTIONAL ANALYSIS, DIFFERENTIAL GEOMETRY AND EIGENVALUES

PREFACE TO THE SELECTED WORKS . . . . .	v
CURRICULUM VITAE . . . . .	xiii
LIST OF PUBLICATIONS . . . . .	xv
SINGER'S WORK ON FUNCTIONAL ANALYSIS, DIFFERENTIAL GEOMETRY AND EIGENVALUES . . . . .	xxv
I. M. SINGER: A BRIEF BIOGRAPHY . . . . .	xxix

### PAPERS

1. with Richard V. Kadison, "Some remarks on representations of connected groups," <i>Proc. Nat. Acad. Sci. U.S.A.</i> <b>38</b> (1952), 419–423. MR 48456 Zbl 0046.25202 . . . . .	<b>1</b>
2. with W. Ambrose, "A theorem on holonomy," <i>Trans. Amer. Math. Soc.</i> <b>75</b> (1953), 428–443. MR 63739 Zbl 0052.18002 . . . . .	<b>7</b>
3. "Automorphisms of finite factors," <i>Amer. J. Math.</i> <b>77</b> (1955), 117–133. MR 66567 Zbl 0064.11001 . . . . .	<b>23</b>
4. with J. Wermer, "Derivations on commutative normed algebras," <i>Math. Ann.</i> <b>129</b> (1955), 260–264. MR 70061 Zbl 0067.35101 . . . . .	<b>41</b>
5. with Richard V. Kadison, "Three test problems in operator theory," <i>Pacific J. Math.</i> <b>7</b> (1957), 1101–1106. MR 92123 Zbl 0078.11503 . . . . .	<b>47</b>
6. with Kenneth Hoffman, "Maximal subalgebras of $C(\Gamma)$ ," <i>Amer. J. Math.</i> <b>79</b> (1957), 295–305. MR 85478 Zbl 0078.11001 . . . . .	<b>53</b>
7. with W. Ambrose, "On homogeneous Riemannian manifolds," <i>Duke Math. J.</i> <b>25</b> (1958), 647–669. MR 102842 Zbl 0134.17802 . . . . .	<b>65</b>

8. “The geometric interpretation of a special connection,” <i>Pacific J. Math.</i> <b>9</b> (1959), 585–590. MR 111062 Zbl 0086.15101 . . . . .	<b>89</b>
9. with Richard V. Kadison, “Extensions of pure states,” <i>Amer. J. Math.</i> <b>81</b> (1959), 383–400. MR 123922 Zbl 0086.09704 . . . . .	<b>95</b>
10. with K. Hoffman, “On some problems of Gel’fand,” in <i>Eighteen Papers on Algebra</i> . American Mathematical Society Translations. Series 2 <b>27</b> (1963). Translation of Russian original published in <i>Uspehi Mat. Nauk</i> 14:3(87) (1959). MR 151866 Zbl 0187.38601 . . . . .	<b>113</b>
11. “Infinitesimally homogeneous spaces,” <i>Comm. Pure Appl. Math.</i> <b>13</b> (1960), 685–697. MR 131248 Zbl 0171.42503 . . . . .	<b>129</b>
12. with Richard V. Kadison, “Triangular operator algebras: fundamentals and hyperreducible theory,” <i>Amer. J. Math.</i> <b>82</b> (1960), 227–259. MR 121675 Zbl 0096.31703 . . . . .	<b>143</b>
13. with J. A. Thorpe, “The curvature of 4-dimensional Einstein spaces,” in <i>Global Analysis: Papers in Honor of K. Kodaira</i> (Princeton University Press, 1969), 355–365. MR 256303 Zbl 0199.25401 . . . . .	<b>177</b>
14. with H. P. McKean, Jr., “Curvature and the eigenvalues of the Laplacian,” <i>J. Diff. Geom.</i> <b>1</b> :1 (1967), 43–69. MR 217739 Zbl 0198.44301 . . . . .	<b>189</b>
15. with D. B. Ray, “ $R$ -torsion and the Laplacian on Riemannian manifolds,” <i>Advances in Math.</i> <b>7</b> (1971), 145–210. MR 295381 Zbl 0239.58014 . . . . .	<b>217</b>
16. with D. B. Ray, “Analytic torsion for complex manifolds,” <i>Ann. of Math.</i> (2) <b>98</b> (1973), 154–177. MR 383463 Zbl 0267.32014 . . . . .	<b>283</b>
17. “Eigenvalues of the Laplacian and invariants of manifolds,” in <i>Proceedings of the International Congress of Mathematicians 1</i> (Canad. Math. Congress, Montreal, Que., 1975), 187–200. MR 420746 Zbl 0345.58014 . . . . .	<b>307</b>
18. with Wong Bun, Shing-Tung Yau and Stephen S.-T. Yau, “An estimate of the gap of the first two eigenvalues in the Schrödinger operator,” <i>Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4)</i> <b>12</b> :2 (1985), 319–333. MR 829055 Zbl 0603.35070 . . . . .	<b>321</b>

APPENDIX

ACKNOWLEDGEMENTS . . . . .	A-1
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**VOLUME 2**  
**INDEX THEORY**

PREFACE TO THE SELECTED WORKS .....v  
 CURRICULUM VITAE..... xiii  
 LIST OF PUBLICATIONS..... xv  
 SINGER'S WORK ON INDEX THEORY .....xxv

PAPERS

19. with M. F. Atiyah, "The index of elliptic operators on compact manifolds,"  
*Bull. Amer. Math. Soc.* **69** (1963), 422–433. MR 157392 Zbl 0118.31203 ..... **337**

20. with M. F. Atiyah, "The index of elliptic operators I," *I. Ann. of Math. (2)*  
**87:3** (1968), 484–530. MR 236950 Zbl 0164.24001 ..... **349**

21. with M. F. Atiyah, "The index of elliptic operators III," *Ann. of Math. (2)* **87**  
 (1968), 546–604. MR 236952 Zbl 0164.24301 ..... **397**

22. "Elliptic operators on manifolds," in *Pseudo-differential operators* (Stresa,  
 Italy, 26 August–3 September 1968). Edited by L. Nirenberg. C.I.M.E.  
 Summer Schools 47. Edizioni Cremonese (Rome), 1968. MR 262712 ..... **457**

23. with M. F. Atiyah, "Index theory for skew-adjoint Fredholm operators,"  
*Inst. Hautes Études Sci. Publ. Math.* **37** (1969), 5–26. MR 285033 Zbl 0194.55503 .... **499**

24. with M. F. Atiyah, "The index of elliptic operators IV," *Ann. of Math. (2)* **93**  
 (1971), 119–138. MR 279833 Zbl 0212.28603 ..... **521**

25. with M. F. Atiyah, "The index of elliptic operators V," *Ann. of Math. (2)* **93**  
 (1971), 139–149. MR 279834 ..... **541**

26. "Future extensions of index theory and elliptic operators," in *Prospects*  
*in Mathematics* (Edited by F. Hirzebruch). Annals of Math. Studies **70**  
 (Princeton University Press, 1971), 171–185. MR 343319 Zbl 0247.58011 ..... **553**

27. "Recent applications of index theory for elliptic operators," in *Partial*  
*Differential Equations* (Edited by D.C. Spencer). Proc. Sympos. Pure Math.  
**23** (Amer. Math. Soc., Providence, R.I., 1973), 11–31. MR 341538 Zbl 0293.58013. . . **569**

28. with M. F. Atiyah and V. K. Patodi, “Spectral asymmetry and Riemannian geometry,” *Bull. London Math. Soc.* **5** (1973), 229–234. MR 331443 Zbl 0268.58010 . . . **591**

29. with M. F. Atiyah and V. K. Patodi, “Spectral asymmetry and Riemannian geometry I,” *Math. Proc. Cambridge Philos. Soc.* **77:1** (1975), 43–69. MR 397797 Zbl 0297.58008 . . . . . **597**

30. with M. F. Atiyah and V. K. Patodi, “Spectral asymmetry and Riemannian geometry II,” *Math. Proc. Cambridge Philos. Soc.* **78:3** (1975), 405–432. MR 397798 Zbl 0314.58016 . . . . . **625**

31. with M. F. Atiyah and V. K. Patodi, “Spectral asymmetry and Riemannian geometry III,” *Math. Proc. Cambridge Philos. Soc.* **79** (1976), 71–99. MR 397799 Zbl 0325.58015 . . . . . **653**

32. “Some remarks on operator theory and index theory,” in *K-Theory and Operator Algebras*. Lecture Notes in Math. **575** (Springer, Berlin, 1977), 128–138. MR 467848 Zbl 0444.47040 . . . . . **683**

33. with Varghese Matha and Richard B Melrose, “The index of projective families of elliptic operators,” *Geom. Topol.* **9** (2005), 341–373. MR 2140985 Zbl 1083.58021 . . . . . **695**

34. with V. Mathai and R. B. Melrose, “Fractional analytic index,” *J. Diff. Geom.* **74:2** (2006), 265–292. MR 2258800 Zbl 1115.58021 . . . . . **729**

35. with V. Mathai and R. B. Melrose, “Equivariant and fractional index of projective elliptic operators,” *J. Diff. Geom.* **78:3** (2008), 465–473. MR 2396250 Zbl 1147.58018 . . . . . **753**

APPENDIX

ACKNOWLEDGEMENTS . . . . . A-1