The Universe that Rings

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by Mark Chieh-Hsing Lee

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Cover image: Pandora's Cluster, deep field image from the NASA James Webb Space Telescope, released February 2023. Slightly cropped. Source: NASA and STScI.

To my daughter Livia and my son Liphan

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Foreword by Shing-Tung Yau[1]

Mark Chieh-Hsing Lee was a classmate of my wife at National Taiwan University, and we have known each other for a long time. He served at NASA for many years and has done a great deal to cultivate space science, garnering numerous awards for his unique insights along the way. What is especially admirable about Mark is that he also used his spare time to write popular science works that are of great use to the general public with a love of science. In 2017, the National Taiwan University Press brought out the original, Chinese-language edition of the current book.[2] A copy of this was presented to me by Chu Kwo-Ray, another former classmate of Mark and also a physicist.

Mark's work has left me in awe. (Not to mention that I could perceive Mark's insights from the book's title alone.) I have been studying the General Theory of Relativity for more than forty years, but this is the first time I've seen a physicist use such objectivity and beautiful language to introduce it. It is not merely about the origin of the theory and its physical implications, for Mark also discusses important contributions which other mathematicians have made to the development of the Theory. For example, Einstein's great work of 1915 was aided by many experts in geometry, which is a fact I have come to deeply appreciate in recent years. In addition to mentioning Grossmann and Hilbert, the book looks back to the nineteenth century to honor Bernhard Riemann, the great mathematician who made an indelible contribution to the concept of space. Before Riemann, there were only three types of space—Euclidean, spherical, and hyperbolic—all three of which could be described by a single coordinate system. Riemann fundamentally changed that concept of space in his famous 1854 paper, which presented an idea that is completely different from the above three—namely that space can exist in isolation and that at a small scale it is similar to Euclidean space. What's more, he showed us that we can use various coordinate systems to describe the characteristics of Riemann space, but that ultimately the meaningful properties of space have nothing to do with the coordinates we choose. This view is very important because it provides an important equivalence principle for the General Theory of Relativity.

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[1] Shing-Tung Yau (丘成桐) is an internationally renowned mathematician, a recipient of the Fields Medal (1982), and a member of the U.S. National Academy of Sciences.

[2] 宇宙的顫抖—談愛因斯坦的相對論和引力波 (*The Universe that Rings: On Einstein's General Relativity and Gravitational Waves*), National Taiwan University Press, 2017. Second edition, 2019.

Riemann defined curvature in the abstract space he introduced, which is employed to describe the gravitational field in the General Theory of Relativity, and the distribution of matter is in turn represented by a part of the curvature. The distribution of matter changes over time, and so does the curvature, and it is these changes that cause the space-time to "ring." Einstein concluded from this that although gravitational waves are weak, they exist. According to Einstein's equation, the gravitational field and the geometry of space-time are inseparable. They are a single entity.

It is worth noting that Riemann himself pointed out that his space was used to understand physical phenomena. He even suggested that the very small and very large units of space should be described in different ways. From the perspective of modern physics, Riemann was already considering possible structures for quantum space! Riemann once considered whether discrete space would help explain this problem. He began his writings at the age of twenty-five and carried on until he died of lung disease at thirty-nine. In the three years before his death, he went to Italy every winter to escape the cold. He influenced a group of Italian and Swiss geometers over the course of these trips, including Christoffel, Ricci, and Levi–Civita. These geometers promoted Riemann's ideas and strictly defined tensors and connections, both of which became indispensable to the General Theory of Relativity and gauge fields. Ricci introduced the Ricci curvature tensor and proved that this tensor could produce one that satisfied the conservation law. This work was produced by geometers in the middle to latter part of the 19th century, and it just so happened to provide the most important tool for the General Theory of Relativity. In 1934, Einstein wrote an article titled "Notes on the Origin of the General Theory of Relativity" (see *Mein Weltbild*, Amsterdam: Querido Verlag), in which he reviewed the mental process of developing his famous theory. The first stage is, of course, the Special Theory of Relativity, and in addition to Einstein himself, the main architects of this theory are Hendrik Lorentz and Henri Poincaré. An extremely important result of this theory is that distance is affected by time. But Einstein had already realized that the Special Theory of Relativity and Newton's Law of Gravitation were incompatible. Corrections needed to be made! Physicists initially did not expect that the concept of space would undergo fundamental changes after Riemann's breakthrough. They were still trying to modify Newton's theory within the framework of three-dimensional space so that it would conform to the newly discovered Special Theory of Relativity within the framework of three-dimensional space. This idea led Einstein astray for three years! When Einstein was studying in Zurich, his mathematics professor, Hermann Minkowski, was as famous as Hilbert and Poincaré. He said that he had a lazy student who had done important work that needed to be explained from a geometric perspective.

In a 1908 paper, Minkowski constructed a four-dimensional space and introduced a metric tensor with reference to Riemann's method that was able to perfectly explain the Special Theory of Relativity. The symmetry group in the Special Theory of Relativity naturally became the symmetry group of Minkowski space. This paper allowed humanity to realize for the first time that we live in four-dimensional space-time. The most important revelation of Einstein's life came in 1908, as this was when he saw the importance of Minkowski's thesis. It is generally believed that Einstein's most important thought this year was his thought experiment. This is important, of course, but I think this is a mental process Einstein was undergoing in an attempt to digest Minkowski's theory! But why is Minkowski's paper so important? The conceptional shift from three-dimensional space to four-dimensional space was not only a great leap forward, but with four-dimensional space in mind, the new conception of gravitational fields had enough room to show its dynamic phenomena. Newton's Law of Gravitation is static: As long as there is a function, it is sufficient to describe the phenomenon of gravity. But Minkowski brought a new viewpoint to light, namely that we need to use a tensor to perfectly describe gravitational fields. Minkowski's tensor perfectly describes the Special Theory of Relativity, but Einstein wanted to go one step further and combine Newtonian mechanics with Minkowski space. The space-time in his imagination needed to be equivalent to Minkowski's tensor in an extremely small range.

At that time, physicists knew nothing about the concept of tensors (in fact, back then, only a small number of geometers understood tensor analysis). Einstein vaguely knew from the principle of equivalence that he needed tools similar to tensors, but he initially wasn't certain about the matter. So he asked his classmate Marcel Grossmann for help, and he finally figured out that the gravitational field should be described by tensor metrics. This tensor is constantly changing in space and time, but at each point it can be approximated by the Minkowski metrics in the first order. Grossmann studied geometry and helped Einstein with math during university. However, the introduction of the concept of a tensor metric was not enough to describe a gravitational field. We still needed to know how to differentiate non-flat space, while at the same time we also needed the result of the differentiation to have nothing to do with the choice of coordinates (which is a requirement of the equivalence principle). This is the connection theory of Christoffel and Levi–Civita. Einstein said in his memoirs on the General Theory of Relativity that this was the first problem he encountered, and he found that it had already been solved by Levi–Civita and Ricci.

Einstein's second problem involved applying this framework to Newton's gravitational equation. Newton's equation is very simple. It has two derivatives of gravitational potential that are equal to the density of matter. At that time, neither Einstein nor Grossmann knew how to differentiate the tensor metric, meaning that the result had nothing to do with the selected coordinates, and indicating that it must be a certain type of tensor. After repeated requests from Einstein, Grossmann reluctantly went to the library and found Ricci's article that discussed tensors. It turned out that Ricci had already shrunk the Riemann's curvature tensor into a symmetrical second-order tensor. This tensor had the same degree of freedom as tensor metric, and it can be regarded as the second differential of tensor metrics. Einstein was ecstatic when he learned about this. He considered this the left-hand part of his equation, with the right-hand part being the tensor of general matter distribution (in a flat space, this tensor has already been developed and matured). Einstein and Grossmann subsequently published two articles in 1912 and 1913 that proposed this equation. This was hard work. But when Einstein tried to solve this equation with an asymptotic method, he was unable to validate the astronomical phenomenon he sought to explain, which proved exceptionally frustrating. In the following days, in an attempt to explain astronomical phenomena, he tried selecting special coordinates, which essentially gave up the precious and concise principle of equivalence. He and Levi–Civita talked this over many times but to no avail. It wasn't until the spring of 1915 when he visited the great mathematician David Hilbert in Göttingen that new developments were made. Hilbert was certainly adept at geometry, but the most important thing about him is that he was the founder of the modern geometric invariant theory. Additionally, he was surrounded by a generation of masters. Besides Felix Klein, who was an expert at categorizing geometry using symmetry groups, Hilbert's student Hermann Weyl was the founder of gauge field theory, and there was also Emmy Noether, the greatest female mathematician in history and just arrived at Göttingen. The Noether current theory, which would be named after her, was being developed at the time. Einstein was indeed lucky enough to be visiting just at the right time! Hilbert discovered the Hilbert action in October of the same year, and from this action it became possible for the correct gravitational equation to be quickly derived. After Einstein learned the news and received Hilbert's postcard, he also quickly obtained his equation, which allowed him to calculate the astronomical problem he had always hoped to solve. Einstein was initially very upset about Hilbert's quick start, but Hilbert soon announced that his work rightly belonged to Einstein, and Einstein's worries subsequently turned into joy. This represents an epoch-making achievement, and later generations of physicists and mathematicians should regard Einstein with the highest respect. That said, I hope people will not forget the contributions made by the group of geometers who helped his cause. Most of what I've said here was included in the article by Einstein, but unfortunately he made no mention of Hilbert's contribution.

It is worth noting that the equation written by Einstein and Grossmann in 1912 is actually correct—provided that there is no distribution of matter. In fact, this equation allowed them to derive a solution to Schwarzchild's famous equation, which was enough for Einstein to work on the astronomical calculations he wanted. From this, we can see the importance of Grossmann's contribution to the General Theory of Relativity. Regrettably, his name has not received the attention it deserves, which I consider unfair. But if there is one fault to Grossmann, it would have to do with his carelessness, for he did not carefully refer to the literature. Ricci had already discovered the left-hand part of Einstein's equation and knew that only such a combination could satisfy the conservation law. The law of conservation is an extremely important law in physics. After all, the right-hand part of the equation already knows that to fulfill the law of conservation, the left-hand part must also satisfy it. Considering this, the equation should have been discovered before 1912. It was not discovered because Einstein did not thoroughly understand that the left-hand side of the 1912 equation was not perfect, and was insistent that explaining physical phenomena was more important than understanding the inner beauty of geometry.

After completing work on the General Theory of Relativity, Einstein changed his perspective and came to believe that at the most basic level, physics should be guided by mathematics and thought experiments. At the end of the article, he said that after discovering the equation for the General Theory of Relativity, everything came naturally, and it all seemed quite simple too. For a capable scholar, working on it was a breeze. But before he found this truth, he worked diligently for years, suffering day and night, and endured hardships not worth mentioning to others.

Einstein's work can be considered the greatest scientific achievement in the history of humanity, and it is expertly narrated by Mark in this book. I am certain it will inspire many with an interest in science, and I would happily recommend it to anyone.

— S.T. Yau

Preface

EINSTEIN'S THEORY OF RELATIVITY is perhaps the most beautiful and profound physics theory in the history of humanity. Unfortunately, there are too few popular science books that give general readers an appropriately comprehensive explanation of it.

The popular science books introducing Einstein's theory of relativity that are currently on bookshelves are in general either too simple or unabashedly esoteric, leaving virtually no middle ground to speak of. The books with simpler content, such as the top ten I have read, including the two editions of Hawking's *A Brief History of Time*, discuss topics that readers have already been aware of for a long time, but without exploring essential related topics with which those readers are probably unfamiliar. As for the books on the esoteric end of the spectrum, they often begin with Einstein field equations. With a little effort, readers can just make it past the first line of the book, but the excess of technical jargon in the second line quickly overwhelms them, extinguishing rapidly their enthusiasm in the topic.

In fact, even popular science books on the theory of relativity that are intended to be simple and easily understood turn out to be just the opposite. The main problem is that when these books introduce the central tenets of Einstein's thinking, they often present them out of context instead of exploring the ideas sequentially according to the overall development of Einstein's thought process. But by presenting the idea systematically and moving from A to Z, readers are most likely able to gain an understanding of key points from each phase of Einstein's thinking, allowing them to progress gradually and in the end obtain the joy and satisfaction of comprehensive understanding.

This line of thought prompted me to write a popular science book that introduces Einstein's theory of relativity from a "middle ground" that is neither superficial nor too deep to be easily understood. I wanted to provide readers who love knowledge, and whom I regard as kindred spirits, with the ability to fully employ specialist terms when discussing the theory of relativity with their own friends in a party setting. Of course, and more importantly, I hoped to share with readers the incomparable satisfaction of the knowledge gained from Einstein's theory of relativity—a crowning achievement of human wisdom.

I believe that, for general readers, understanding Einstein's theory of relativity from a high-level, scientific perspective is not an impossible task to accomplish.

I had previously always considered Einstein's four-dimensional, curved geometric space-time theory to be, mathematically speaking, overly convenient and simplistic, and I never really believed that there was any chance it represented the actual structure of our universe. However, the gravitational waves recently detected by humanity prove that Einstein's four-dimensional space-time is just as real as a hard metal or diamond that you can touch and feel with your hands.

A hundred years ago, humanity believed the universe was static; but then, over the span of sixty years, humanity's understanding of the universe rapidly grew. Today in the twenty-first century, it is believed that the universe is accelerating with increasing speed as it is being pushed by dark energy. Although Einstein's theory of relativity emerged during the era when people believed the universe to be static, today, one hundred years later, we have actually come to embrace the notion that our universe is controlled by dark energy and dark matter, and that its rate of expansion is accelerating. Indeed, Einstein's theory of relativity is truly amazing. After being rigorously tested for over a century, it remains magnificent and ever-refreshing.

On 14 September 2015, humanity directly detected the gravitational wave that Einstein had predicted a hundred years earlier. The magnitude of the gravitational wave was actually just a faint tremor of the universe, but its power affected me so greatly that I was inspired to write this book. The discovery of direct observation of gravitational waves was expeditiously awarded the 2017 Nobel Prize in Physics.

— Mark Lee