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Essays on Geometric Flows

Celebrating 40 Years of Ricci Flow

Editors

Huai-Dong Cao · Richard Hamilton
Shing-Tung Yau

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