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Geometry of space, physics and analysis*

by Shing-Tung Yau†

We must admit with humility that, while number is purely a product of our minds, space has a reality outside our minds, so that we cannot completely prescribe its properties a priori.

C. F. Gauss (Letter to Bessel, 1830)

The concept of space has gone through many stages of evolution. Many of them are related to the advancement of our understanding of nature.

From Ancient to Modern Geometry

In the days of the Greek geometers, plane and Euclidean geometry were reasonably adequate to describe most observations. The Greek scholars were convinced that the earth is round and were able to measure the diameter of the earth and its distance to the sun based on plane geometry. The Chinese scholars also measured the distance of the earth to the heaven using similar ideas.

But even then, Archimedes had already started to investigate infinite processes to understand geometric figures that could not be described by Euclidean geometry alone. Calculations of volumes of many different geometric figures started the investigation of integral calculus, as was also proposed by ancient Chinese mathematicians.

Not long afterwards, in the process of measuring the movement of celestial bodies, the ancient Indians and Chinese discovered the concept of differentiation of trigonometric functions.

Copernicus proposed that the sun, instead of the earth, is the center of the universe. The work of Kepler and Galileo that supported Copernicanism appeared a half century later. It was widely accepted only after Newton formulated the universal law of gravitation.

When Isaac Newton merged the development of differential and integral calculus together and verified Kepler's laws of motion of planets, mathematics reached a new plateau, largely because calculus gave us a most powerful tool to understand and calculate the geometry of curved objects that appear in nature.

Some historians said that Leibniz independently discovered calculus. Many of the mathematical notations we used today are in fact due to him. However, without Newton's important application of calculus to mechanics, few people would have paid much attention to the development of calculus.

Soon afterwards, many great mathematicians developed new disciplines of mathematics based on this new tool of calculus. The most notable was the development of differential geometry and the introduction of calculus of variations. Fermat, Euler and Monge are among the most notable contributors. Some of their works are still being investigated today. It is amazing that even then, calculus of variations was used extensively by differential geometers.

In his famous paper *On the Hypotheses Which Lie at the Foundation of Geometry (1854)*, Riemann wrote: "The theorems of geometry cannot be deduced from the general notion of magnitude alone, but only from those properties which distinguished space from other conceivable entities, and these properties can only be found experimentally … This takes us into the realm of another science – physics."

* This article is based on the author's talk delivered at Tsinghua University in the summer of 2015, on the hundredth anniversary of the discovery of general relativity. The author's works on geometry and physics are collected in a two-volume set, *Selected Expository Works of Shing-Tung Yau with Commentary*, International Press, 2014.

† Department of Mathematics, Harvard University, Cambridge, MA 02138, USA
E-mail: yau@math.harvard.edu